Unitary Qubit Lattice Simulations of Multiscale Phenomena in 2D and 3D Quantum Turbulence

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A unitary qubit lattice algorithm, which scales almost perfectly to the full number of cores available (e.g., 216000 cores on a CRAY XT5), is used to examine quantum turbulence and its interrelationship to classical turbulence with production runs on grids up to 5760³. The maximal grids achievable by conventional CFD for quantum turbulence is just 2048³, and artificial dissipation had to be introduced to suppress high-k modes but destroyed the Hamiltonian system. Our unitary algorithms preserve the Hamiltonian structure of the Gross-Pitaevskii equation which describes quantum turbulence in a zero-temperature Bose-Einstein condensate (BEC). As a result, parameter regimes have been uncovered which exhibit very short Poincare recurrence time, as well as a strong triple cascade structure in the kinetic energy spectrum. For small-k, the total energy spectrum exhibits a Kolmogorov k-5/3 spectrum -- associated with large scale effects on which vortex quantization is unimportant. The incompressible spectrum exhibits a large-k k-3 spectrum but adjoined by a k-4 Saffman-like spectrum reminiscent of vorticity discontinuities. Both 2D and 3D quantum turbulence are examined.

A quantum vortex is a topological singularity, with zero density at the core. Renormalized vorticity and enstrophy in 2D pinpoint the vortex core location, which has properties similar to a branch cut and its associated branch points. As opposed to dissipative 2D Navier-Stokes turbulence, in which like-rotating vortices coalesce, in 2D quantum turbulence one has creation/annihilation of counter-rotating vortices. Thus enstrophy is no longer conserved as in classical 2D turbulence. As a result, we do not observe inverse cascade of energy in quantum turbulence.

The unitary qubit algorithm involves a complex series of interleaved unitary collision operators that locally entangle the on-site qubits which are then streamed to nearby lattice sites. The 3D vorticity isosurfaces can be generated during the production run itself as they require just 512 cores for data generated on a 4800³ lattice (2 minutes/time step on a CRAY XE6)

A simpler unitary qubit algorithm has also been developed for the 1D MHD-Burgers model of Diamond.

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