SEMI-LAGRANGIAN VLASOV SOLVERS

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Abstract

Cheng and Knorr [1] introduced a method for solving the Vlasov equation on a grid of phase space. It was based on a time splitting between position and velocity coordinates, so that each split step consisted in a constant coefficient advection solved using a shift and a cubic spline interpolation. This method was recognized to be a special case of the more general semi-Lagrangian method widely used in climate simulations [6]. This original method which we now call backward semi-Lagrangian method consists in two steps: 1) backtrack the position of the particle ending at each phase space grid point, 2) Interpolate the particule distribution function at the obtained position to get its new value. This method can be used with or without time-splitting. This method has proved very efficient for many plasma physics problems. Let us cite in particular its application for the gyrokinetic Vlasov equation in the GYSELA code [5].

However this method has a few drawbacks. It is not exactly conservative and the computation of the origin of the characteristics is implicit in some cases, which makes it harder using high order time stepping schemes. A conservative Vlasov solver has been introduced in [4] and has been discovered to be a special case of a form of conservative semi-Lagragian solver [2]. On the other hand a class of forward semi-Lagrangian solvers has been introduced in [3], which has the advantage of being both exactly conservative, explicit and very close to traditional Particle In Cell solvers. Those methods have also been implemented on mapped meshes which allow the representation of complex geometries like Tokamaks and a moving grid.

In this talk we are going to present and compare the different kinds of semi-Lagrangian Vlasov solvers and give an overview of the recent results on this subject.

References

[1] C. Z. Cheng, G. Knorr, The integration of the Vlasov equation in configuration space, J. Comput. Phys, **22**, pp. 330-3351, (1976).

[2] N. Crouseilles, M. Mehrenberger, E. Sonnendrücker, Conservative semi-Lagrangian schemes for the Vlasov equation, J. Comput. Phys. **229**, pp. 1927-1953, (2010)

[3] N. Crouseilles, T. Respaud, E. Sonnendrücker (2009): A forward semi-Lagrangian method for the numerical solution of the Vlasov equation, Comput. Phys. Comm., 180, pp. 1730-1745.

[4] F. Filbet, E. Sonnendrücker, P. Bertrand, Conservative numerical schemes for the Vlasov equation, J. Comput. Phys., **172**, pp. 166-187, (2001).

[5] V. Grandgirard, M. Brunetti, P. Bertrand, N. Besse, X. Garbet, P. Ghendrih, G. Manfredi, Y. Sarrazin, O. Sauter, E. Sonnendrücker, J. Vaclavik, L. Villard, A drift-kinetic semi-Lagrangian 4D code for ion turbulence simulation, J. Comput. Phys., **217**, pp. 395-423, (2006).

[6] E. Sonnendrücker, J. Roche, P. Bertrand, A. Ghizzo The semi-Lagrangian method for the numerical resolution of the Vlasov equation, J. Comput. Phys., **149**, pp. 201-220, (1999).