Numerical Aspects of Drift Kinetic Plasma Turbulence: Ill-posedness, Regularization and Apriori Estimates of Sub-grid-scale Terms

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It is claimed that the Vlasov-Poisson system of equations governing drift-kinetic plasma turbulence is ill-posed in the sense of Hadamard. For example, it is known in gyrokinetic plasma turbulence, nonlinear interactions in velocity space lead to generation of small scales of the distribution function right up to the collisional scales (see [Schekochihin 2008]). Without the physical cut-off mechanisms provided by collisions or some form of numerical regularization of the Vlasov-Poisson system, it is shown that the distribution function develops small scale features which may lead to a lack of convergence in numerical simulations. In this work, we present an alternative numerical method to the semi-Lagrangian approach [Grandgirard 2006] for solving the Vlasov-Poisson system for 4D drift kinetic turbulence. Our numerical approach uses a conservative formulation with high-order (fourth and higher) evaluation of the numerical fluxes coupled with a fourth-order accurate Poisson solver. The fluxes are computed using a low-dissipation high-order upwind differencing method [Pirozzoli 2002] or a tuned high-resolution finite difference method with no numerical dissipation [Hill 2004]. Numerical results are presented for the case of imposed ion temperature and density gradients. Different forms of controlled regularization to achieve a well-posed system are used to obtain convergent resolved simulations. The regularization of the equations is achieved either by means of a simple collisional model [Rathman 1975] or by inclusion of an ad-hoc hyperviscosity term.

The notions of direct numerical simulation (DNS) and large-eddy simulation (LES) are well established in hydrodynamic turbulence (see, for example, Reference [Lesieur 1996]). Similar to the filtering formalism applied to the Navier-Stokes equations, we apply a filter to the Vlasov equation and derive terms analogous to the Reynolds stress terms. To the best of our understanding, the contributions of these SGS terms are ignored in under-resolved simulations of plasma turbulence utilizing an Eulerian or semi-Lagrangian approach wherein the entire 4D (or 5D in gyrokinetics) space is meshed. Furthermore, analogous to the approach in hydrodynamic turbulence, we present *apriori* quantifications of these SGS terms in resolved simulations of drift-kinetic turbulence (here "*apriori*" is used in distinction from "*aposteriori*" quantifications of SGS terms). Contributions of the SGS terms is quantified by applying a sharp filter to the resolved simulations, and are undertaken with a view to understand the relative importance of the SGS terms compared with the nonlinear terms above the filter cut-off. Finally, we believe that such detailed quantifications may lead to the development of phenomenological models of SGS terms to be utilized in under-resolved plasma turbulence simulations.

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