A Fully Implicit Newton-Krylov-Schwarz Method for Tokamak MHD: Jacobian Construction and Preconditioner Formulation

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Single-fluid resistive magnetohydrodynamics (MHD) is a fluid description of a plasma which is frequently employed to investigate large scale macroscopic instabilities in tokamaks. In MHD modeling of non-spherical tokamaks it is often desirable to compute MHD phenomena to resistive time scales or a combination of resistive-Alfvén time scales which make explicit time stepping schemes computationally expensive. We present recent advancements in the development of preconditioners for fully nonlinearly implicit simulations of single-fluid resistive tokamak MHD. Our work focuses on simulations using a structured mesh mapped into a toroidal geometry with a shaped poloidal cross-section, and a finite volume discretization of the spatial derivatives in the visco-resistive MHD model. We discretize the temporal dimension using a fully implicit θ-method in time. We solve the resulting nonlinear algebraic system using a standard inexact Newton-Krylov approach, through the SUNDIALS library. In this solver, all Jacobian-vector products are computed in "matrix-free" mode, using the underlying physical flux routines to generate finite-difference approximations to the directional derivatives used within the solver. The basic approach for all of these algorithms has been previously described in [2, 3].

As is well-understood with inexact Newton methods, true algorithmic scalability with increasing problem size requires an effective preconditioner for accelerating the convergence of the inner Krylov solver. For the MHD equations considered, effective preconditioners require information about the Jacobian entries, to be used within some approximate solution strategy. Analytical formulae for these Jacobian entries may be difficult to derive and implement without error, especially so if nonlinear upwind methods are used. To this end, we have employed the OpenAD tool, one feature of which is that it reads in highly modularized Fortran90 code as input, and outputs new Fortran90 source code that implements the Jacobian of the input subroutine. This pre-processing only needs to occur once, and not at each step of a simulation. Moreover, the resulting code is accurate to full double precision, and not just the 6 or 7 digits available through approximate difference calculations of the Jacobian.

Given these Jacobian entries, we investigate three preconditioning approaches: a 3D restricted additive Schwarz (RAS) [1] approach using SuperLU for processor-local linear solves, a 3D block ADI preconditioner (similar to that used in Reference [3]) that solves parallel periodic, block-tridiagonal linear systems along each coordinate direction, and a hybrid approach that employs a 2D restricted additive Schwarz solver within each poloidal plane, and a parallel periodic, block-tridiagonal solver in the toroidal direction. We present numerical results for each approach outlined above in the context of pellet-injection fueling of tokamak plasmas. We also describe the code modifications necessary for use of the OpenAD tool and SUNDIALS solver library.

References

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