Quasi Monte Carlo simulation of fast ion thermalization

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This work investigates the applicability of quasi-random sequences in the Monte Carlo method for simulation of fast ion thermalization processes in plasmas, e.g. neutral beam injection [1, 2] and RF-heating [3]. Quasi-random sequences [4] are purely deterministic with the special property of high uniformity over the d-dimensional unit cube and with weak convergence close to $\mathcal{O}(N^{-1})$, where N is the number of markers. This is a significant improvement of the convergence compared to the standard Monte Carlo, which is $\mathcal{O}(N^{-1/2})$. It has been shown in [5] that a straightforward use of quasi-random sequences for simulation of stochastic differential equations will either lead to a systematic error or similar weak convergence as the standard Monte Carlo method. This is caused by the internal correlation in these sequences and on breakdown of uniformity if very high dimensions are considered. For pathwise simulations where the number of dimension is proportional to the number of time steps, breakdown of uniformity can be circumvented by using a so called Brownian bridge discretization [6] of the underlying Brownian motion. When entire ensemble of particles are considered, a low-dimensional sequence can be used, and the correlation can be broken by sorting the particles each time step [7]. In this work we have compared these two methods for simulation of fast ion thermalization processes. Numerical convergence of the two methods is tested on a simplified simulation of the energy scattering process [8] and on a simulation of neutral beam injection.

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