Hybrid codes: numerical properties and applications to astrophysical shock scenarios

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Hybrid codes, where ions are treated kinetically and electrons are a charge neutralizing fluid, share properties of both fully kinetic particle-in-cell (PIC) codes and fluid codes. In order to study kinetic plasma effects on the time-scale of the inverse ion cyclotron frequency, ω_{ci}^{-1}, the displacement current in Ampère's law is neglected, and the electrons are treated as a massless charge-neutralizing fluid. The resulting system of equations can be solved explicitly neglecting the electron-scale frequencies, and also the ion plasma frequency, which then allows us to evolve a physical system in time much more efficiently. This kind of approach is extremely useful in the study of non-relativistic magnetized astrophysical shocks, among other plasma physics problems.

The two main numerical difficulties in using hybrid codes are the numerical noise usually associated with PIC codes, due to the discretization of the fields on a grid and the use of finite number of particles, and the dependency of the stability of the algorithm on the local plasma density. The numerical noise problem can be solved by either increasing the number of particles per cell in a simulation, or by using numerical filtering techniques. The second problem is more intricate and can also be translated (in first order of approximation) in the statement that the Alfvén velocity $V_A = B_0/\sqrt{4\pi mn}$ depends inversely on the plasma density, and hence increases when $n \to 0$.

In this work we first conduct a quasi-linear analysis of the hybrid set of equations and compare the hybrid and fully kinetic wave dispersion equations. This analysis shows the domain of validity of the hybrid approach, constrains the choice of numerical parameters to be used in hybrid simulations, and clearly shows why hybrid codes are unstable in the $n \to 0$ limit. We then present a complete implementation of the hybrid method in our code *dHybrid*, a parallel 3D non-relativistic code. Here we show that it is crucial to time- and space-center all numerical calculations, and we present a novel method to accomplish this task; in particular, we present a semi-implicit algorithm that is intrinsically stable for low plasma density conditions. Energy conservation in our case is typically better than for other hybrid codes described in the literature, at the expense of computational time, which then allows us to analyze physical systems over very large time intervals $T \sim 2000\,\omega_{\rm ci}^{-1}$.

We simulate a non-relativistic magnetized shock with *dHybrid*, and compare the results with a fully kinetic PIC simulation performed with the code *Tristan-MP*. Results are similar for the two codes over the physical time-span that can be probed by both models, which validates the use of our numerical algorithm in *dHybrid*. Finally, we analyze the evolution in time of the shock structure, and the late-time properties of the accelerated particle spectra using the hybrid model, and draw conclusions on the applicability and limitations of hybrid codes to these physical scenarios.