Mesh adaptation and Monge-Kantorovich optmization

G.L. Delzanno[1], L. Chacon[2], J.M. Finn[1]
[1] Los Alamos National Laboratory
[2] Oak Ridge National Laboratory

Equidistribution of a monitor function or an error estimate has been a guiding principle in mesh generation for a long time. However, in two and three dimensions (2-3D), an infinite number of meshes can satisfy a given equidistribution principle, implying that there is room for optimization.

In this context, Monge-Kantorovich (MK) optmization choses one equidistributed mesh by minimizing the L_2 norm of the grid point displacement of such mesh relative to an initial mesh [1]. This procedure gives rise to a very robust and efficient multidimensional mesh generation/adaptation method, based on the solution of the nonlinear Monge-Ampere equation. The latter is solved with the multigrid preconditioned Jacobian Free Newton-Krylov method, delivering a scalable algorithm under grid refinement [1].

We will present mesh quality comparisons against alternative equidistribution methods for 2D static problems, showing that MK optimization produces good quality grids with nearly minimal grid cell distortion [1,2]. It is also very robust against mesh tangling, mantaining good quality grids when other competitor methods fail [1]. We will also present our successful application of MK optimization to 3D time-dependent mesh adaptation [3], illustrated with a hyperbolic PDE, the advection of a passive scalar, in 2D and 3D. Velocity flow fields with and without flow shear are considered, demonstrating the optimality of the approach.

[1] G.L. Delzanno, L. Chacon, J.M. Finn, Y. Chung, G. Lapenta, *An optimal robust equidistribution method for two-dimensional grid adaptation based on Monge–Kantorovich optimization,* J. Comput. Phys. 227, 9841 (2008).
[2] G.L. Delzanno, J.M. Finn**,** *Generalized Monge-Kantorovich optimization for grid generation and adaptation in Lp,* SIAM J. Sci. Comput. 32 (6), 3524 (2010).
[3] L. Chacon, G.L. Delzanno, J.M. Finn, *Robust, multidimensional mesh motion based on Monge-Kantorovich equidistribution*, J. Comput. Phys. 230, 87 (2011).