Kolmogorov Flows in Strongly Coupled Plasmas: "A Molecular Dynamics Study"

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The Big Question

Is it possible to investigate large scale hydrodynamic flows in a complex plasma using exact molecular dynamics (MD) simulations ?

Abstract

Using MD simulations, we report for the first time, the fate of large scale Kolmogorov flows in a strongly coupled complex plasma. The prototype system is a two-dimensional (2D) Yukawa liquid. Such flows have been widely used to study flow instability and transition to turbulence in Navier-Stokes fluids [1]. In the 2D case, a Kolmogorov flow is generated by imposing a unidirectional force with magnitude varying sinusoidally along the other direction. For small enough forcing magnitude, this results into a series of parallel shear bands having sinusoidal velocity profile. At higher forcing magnitudes, the destabilizing inertial effects dominate over the stabilizing viscous effects and the flow undergoes a transition to a vortex lattice [2] and eventually turbulence [3].

The State of Strong Coupling

Complex plasma is made up of dust grains, ions, electrons and neutral gas. Grains interact via Yukawa potential:

$$U(r) = \frac{Q^2}{4\pi\epsilon_0 a}\frac{e^{-r/\lambda_D}}{r} \qquad (1)$$

where, Q, a and λ_D are the dust charge, Wigner Seitz radius and Debye length respectively. Yukawa liquids exist in a state of strong coupling where

$$\Gamma = \frac{Q^2}{4\pi\epsilon_0 a k_B T_D} > 1 \qquad (2)$$

where T_D is the dust temperature.

Parameter List

Number of grains: $N = 2.304 \times 10^5$
Size of the system (in units of a): $L = 480$
Number Density (in units of a^{-1}): $n = 1$
Screening parameter: $\kappa = a/\lambda_D = 0.5$
CPU's used: 24. Typical wall time: 48 hrs.

Past Work: Kelvin Helmholtz instability in strongly coupled Yukawa liquid was reported using "first principles" MD simulations [4, 5] as an "initial value problem".

Present Work: In contrast to these works on parallel shear flows [4, 5], Kolmogorov flows is a driven problem and hence fundamentally different in nature.

Equation of motion for grains: $m\ddot{\boldsymbol{r}}_i = -\frac{Q^2}{4\pi\epsilon_0}\sum_{j\neq i}^{N}\nabla U(r_{ij}) + F_0\cos(2\pi n_x x/L)[1+\Delta\cos(2\pi n_y y/L)]\hat{\boldsymbol{y}}$.

Kolmogorov Flows (Driven Problem)

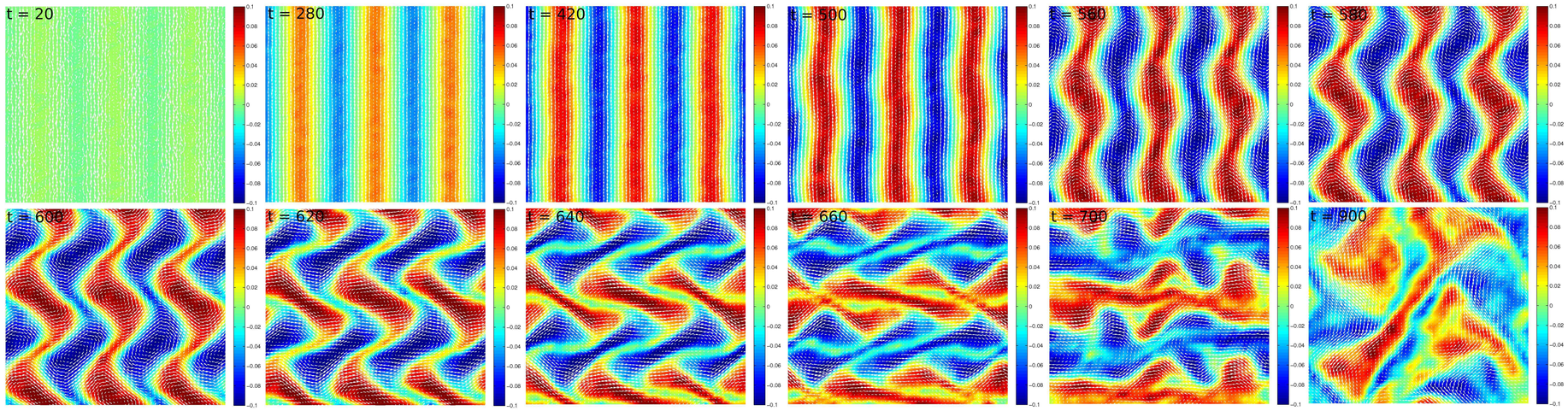

Figure 1: Time evolution of vorticity ($\boldsymbol{\omega} = \nabla \times \boldsymbol{v}$) field in a Kolmogorov flow starting from an initial value of $\Gamma = 120$. Magnitude of forcing $F_0 = 0.005$ and amplitude of perturbation $\Delta = 0$. Grain velocities in the region are fluidized through a 45×45 grid to construct local vorticity. Arrows indicating direction of local velocity are obtained by fluidizing the grain velocities over a 60×60 grid.

Growth rates

Growth rate of the instability can be obtained by plotting time evolution of the perturbed kinetic energy along $\hat{x}$, normalized to its initial value:

$$|\delta E_k^2| = \frac{\int\int v_x(t)^2 dxdy}{\int\int v_x(0)^2 dxdy} \qquad (3)$$

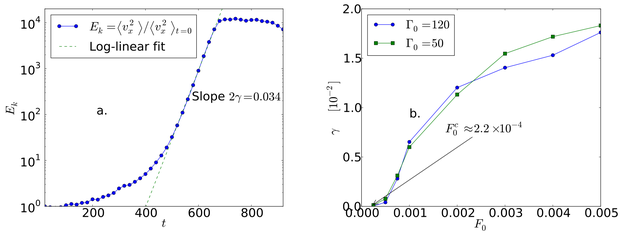

Figure 2: (a.) Time evolution of perturbed kinetic energy shown in Equation (3). The logarithm of $|\delta E_k^2|$ grows linearly in time leading to nonlinear saturation at late times. (b.) Growth rates γ as a function of forcing magnitude F_0 for two values of Γ.

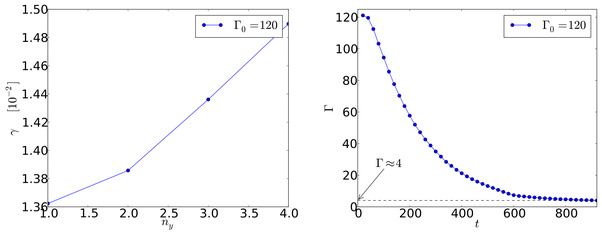

Figure 3: (a.) Growth rate spectra: Initial $\Gamma = 120$ and mode amplitude $\Delta = 0.1$. (b.) Plot of Γ as a function of time corresponding to Figure 1. The rise in temperature is clearly seen due to shear induced heating [5].

Conclusions

- We performed MD simulation of Kolmogorov flows starting from two values of $\Gamma = 120$ and 50.
- Flows are stable upto a critical $F_0^c \approx 2.2 \times 10^{-4}$.
- Amplitude of driving force F_0^c is seen to be roughly the same for both the $\Gamma = 50$ (driven problem).

Future Work

- Linear stability analysis using a generalized hydrodynamic model.
- Comparison of present results with thermostatted (constant Γ) runs.

References

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