Efficient Hybrid Methods for the Simulation of Plasmas with Coulomb Collisions

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UCLA & LLNL

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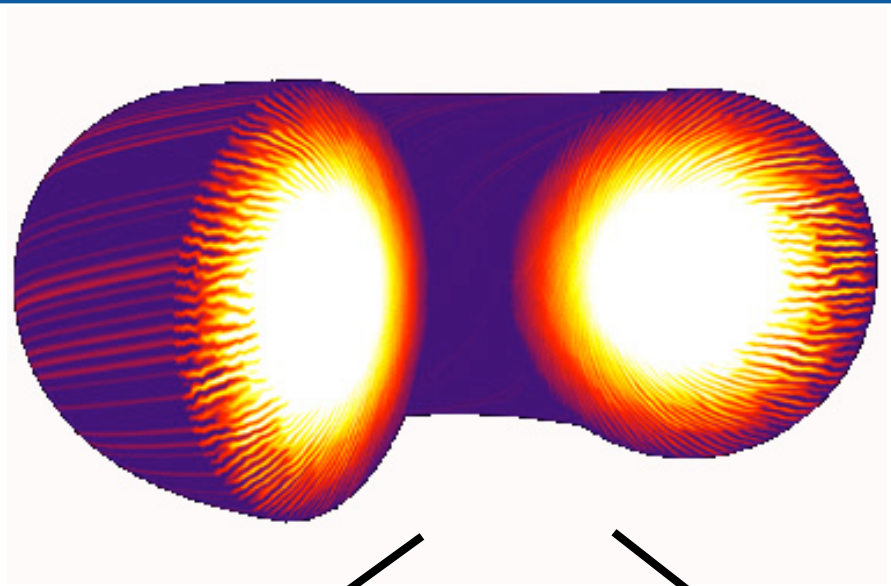
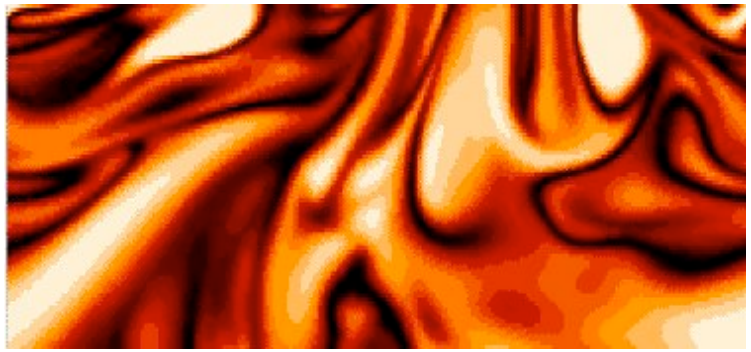
Introduction: $Kn \equiv \lambda_{\mathrm{mfp}}/L \lesssim 1$





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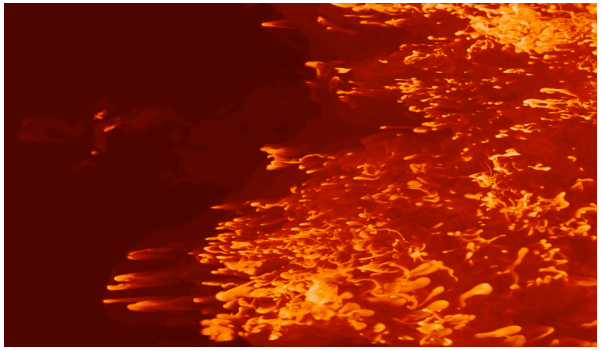

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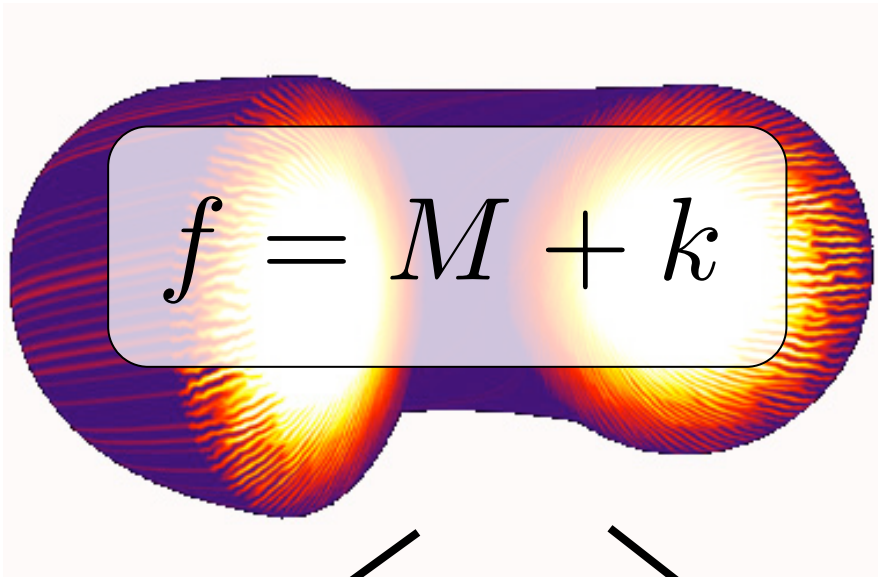

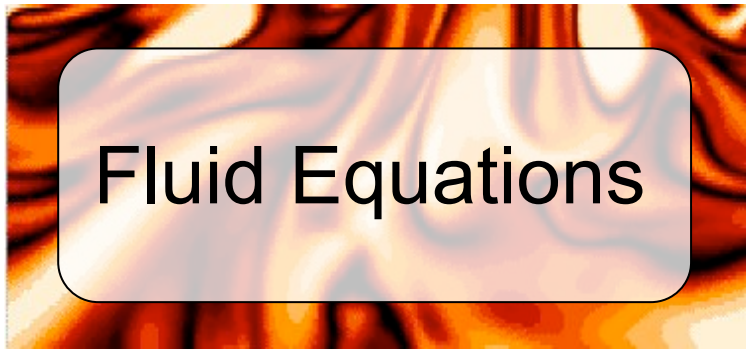

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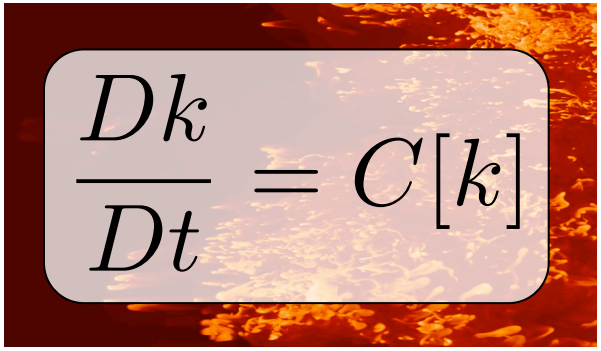

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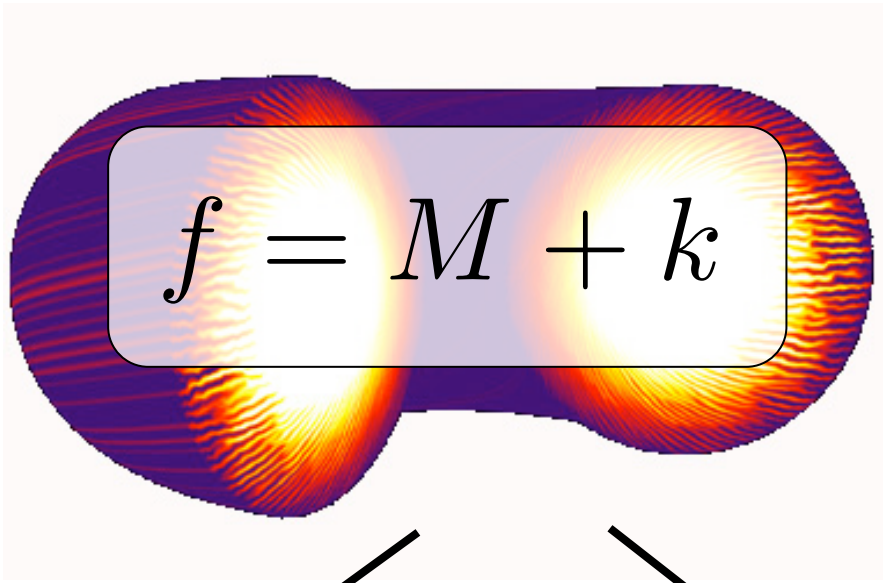
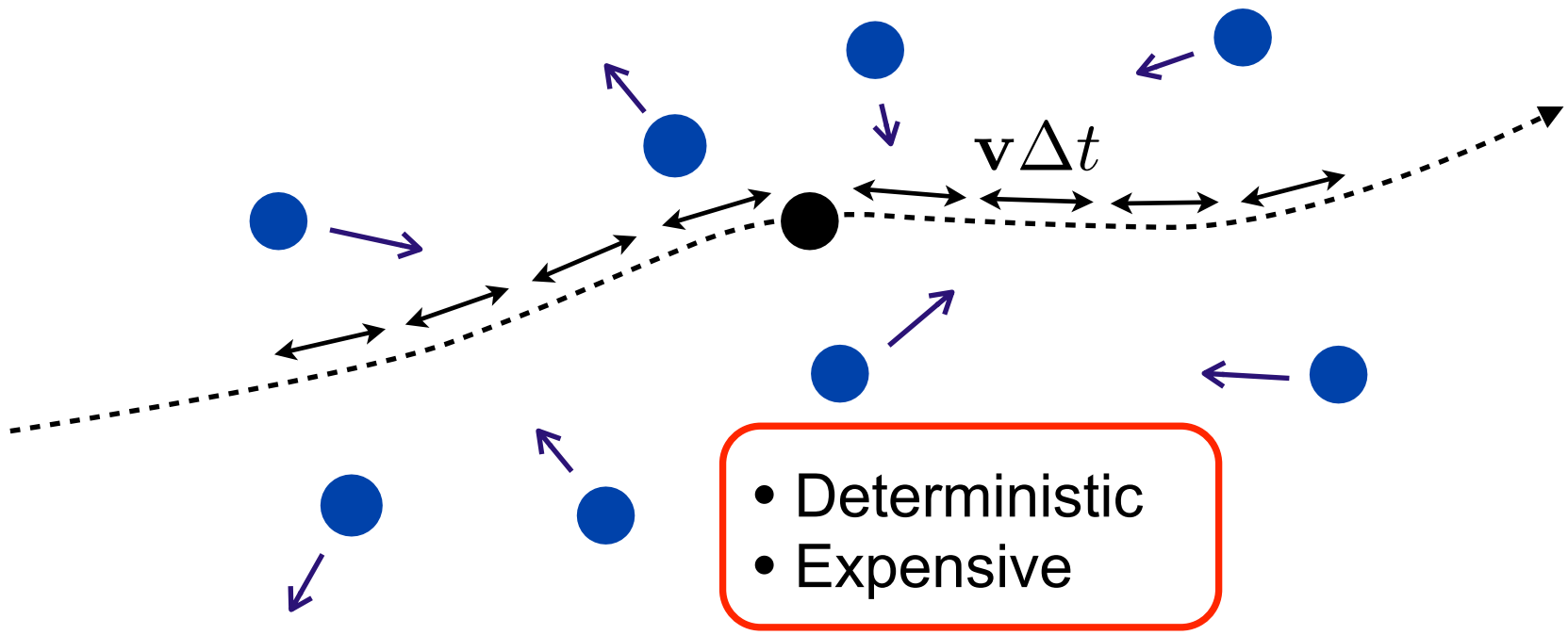


Fluid Equations

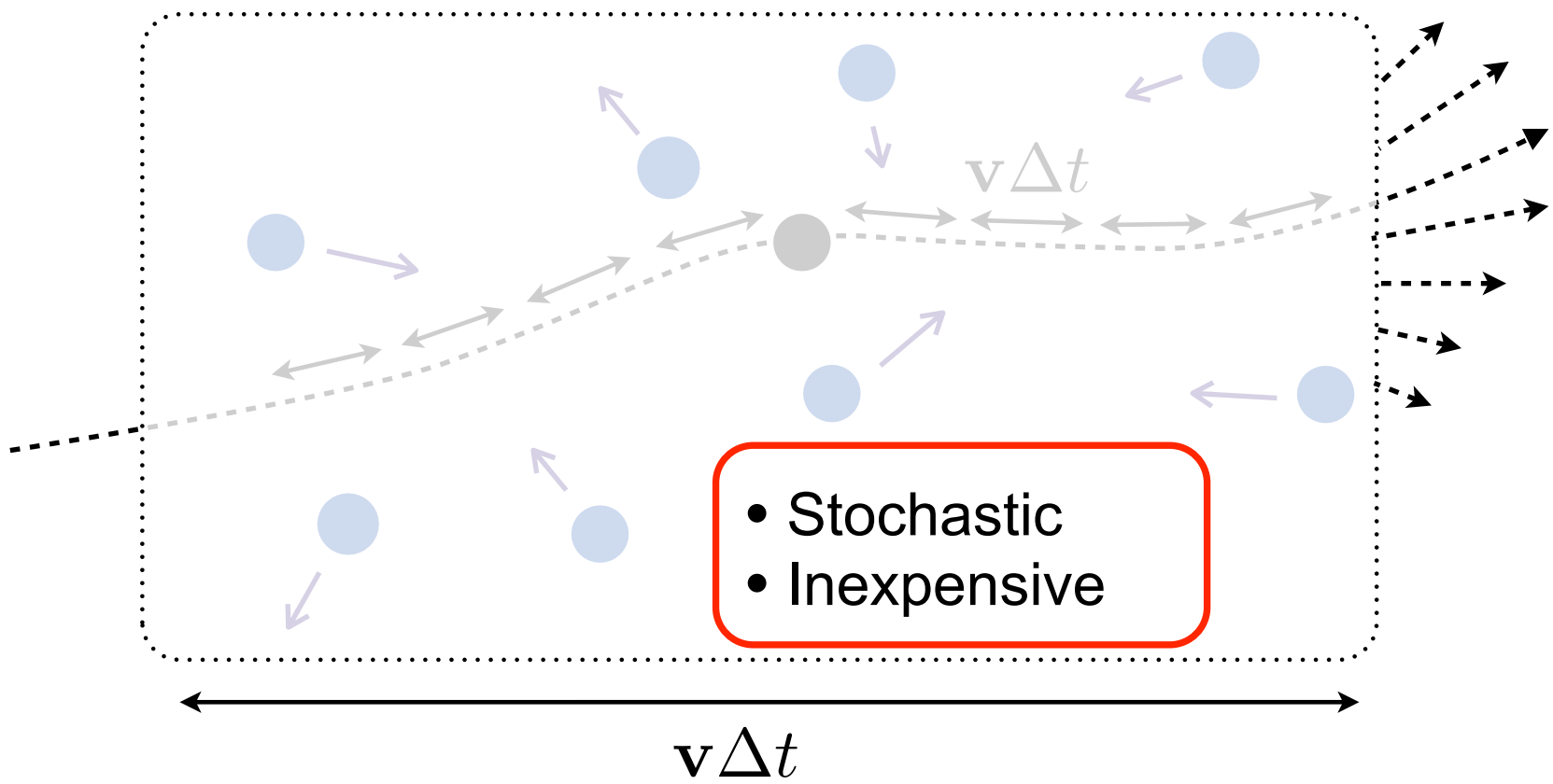
$$\frac{Dk}{Dt} = C[k]$$

Collisions: Binary Coulomb



- Deterministic
- Expensive

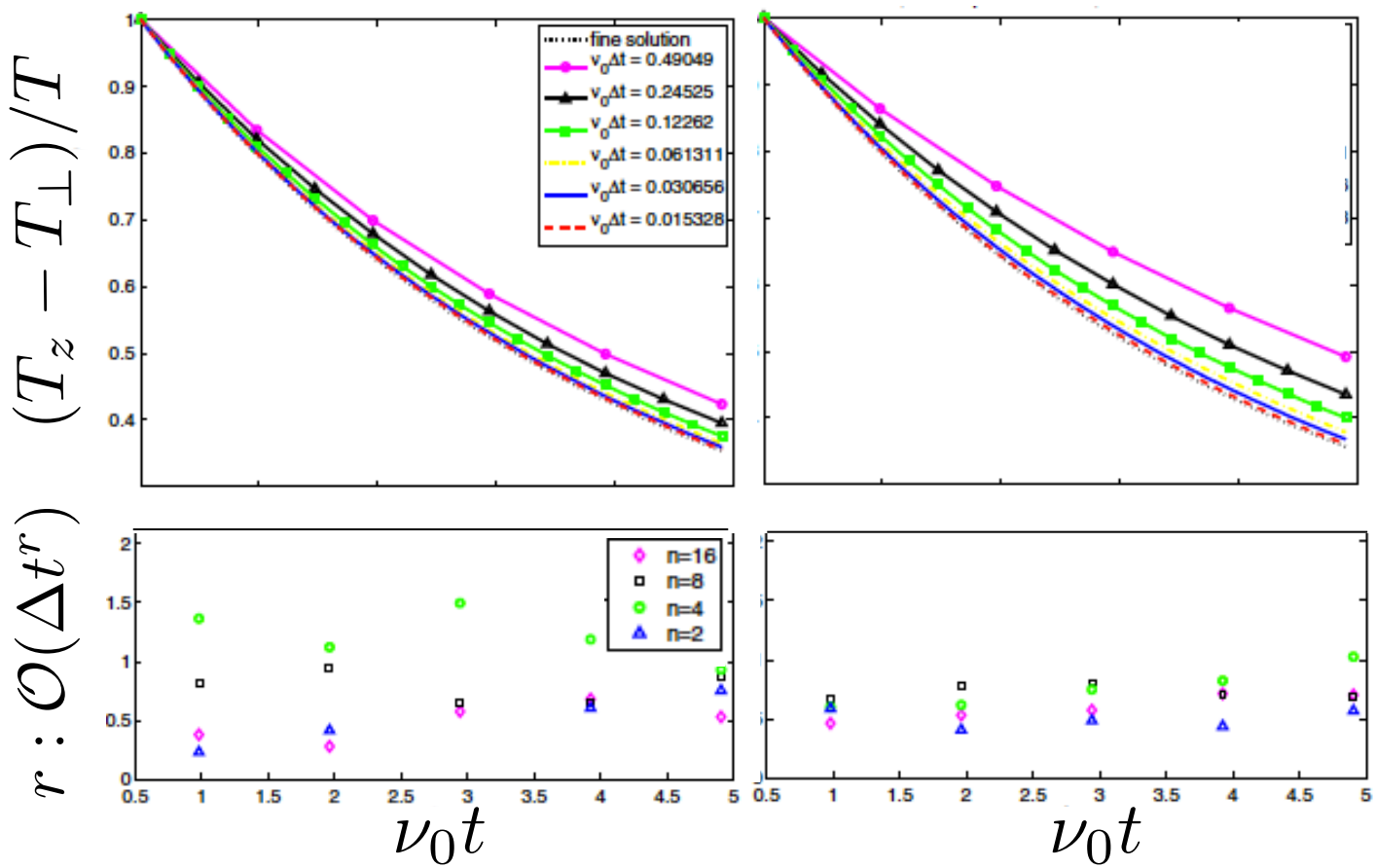
Collisions: Binary Coulomb



1. Takizuka, Abe, 1977, *J. Comp. Phys*, 25, 205
2. Nanbu, 1997, *Phys. Rev. E.*. 55, 4642

Collisions: Binary Electron-Electron Test Case

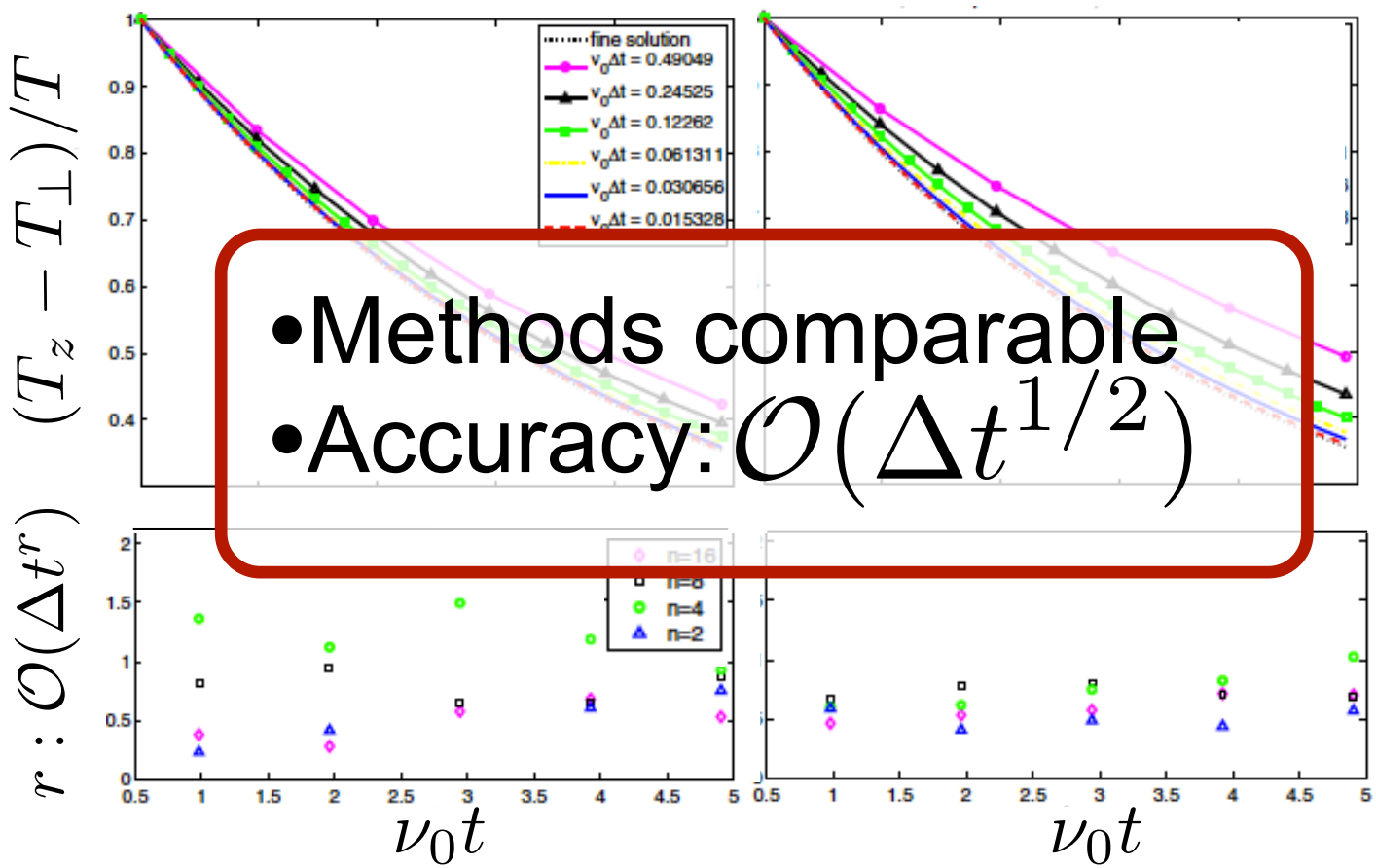
Nanbu Takizuka & Abe



1. Wang, Lin, Caflisch, Cohen, Dimits, 2008, *J. Comp. Phys,,* 227, 4308
2. Dimits, Wang, Caflisch, Cohen, Huang, 2008, *J. Comp. Phys.,* 228, 4881

Collisions: Binary Electron-Electron Test Case

Nanbu | Takizuka & Abe



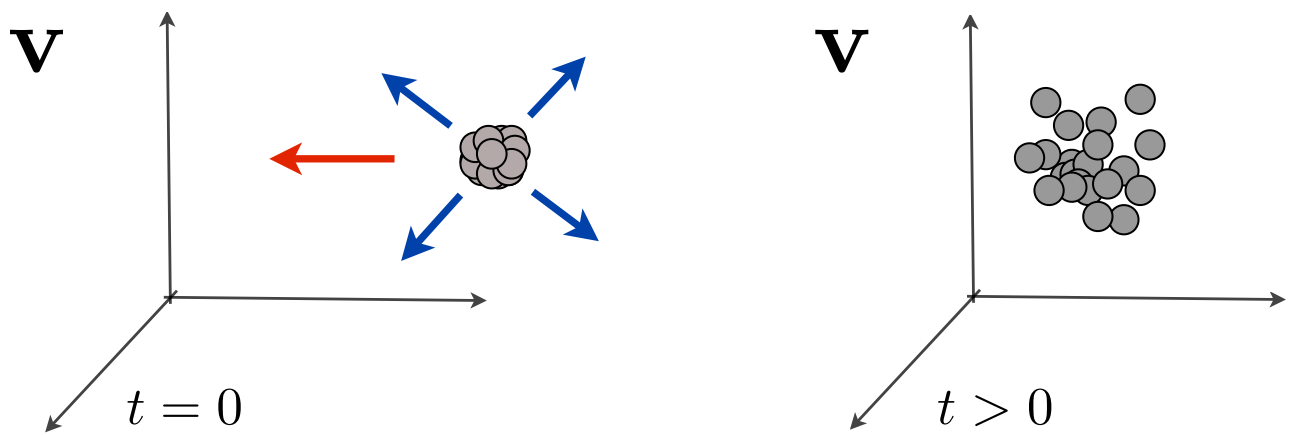
- Methods comparable
- Accuracy: $\mathcal{O}(\Delta t^{1/2})$

1. Wang, Lin, Caflisch, Cohen, Dimits, 2008, *J. Comp. Phys,,* 227, 4308
2. Dimits, Wang, Caflisch, Cohen, Huang, 2008, *J. Comp. Phys.,* 228, 4881

Collisions: Stochastic Differential Equation

Drag Diffusion

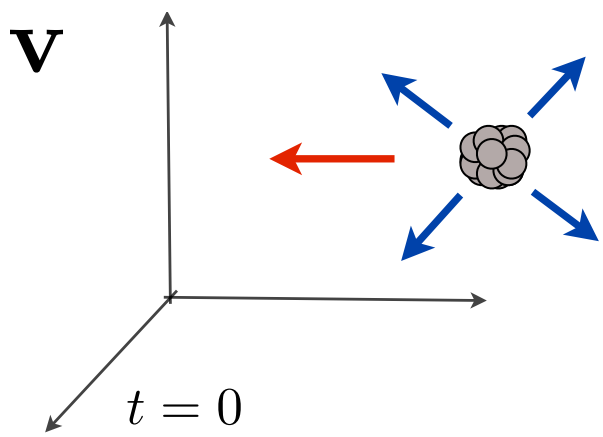
$$C[f] = -\frac{\partial}{\partial \mathbf{v}} \mathbf{F_d} f + \frac{\partial^2}{\partial \mathbf{v} \partial \mathbf{v}} : \mathbf{D} f$$

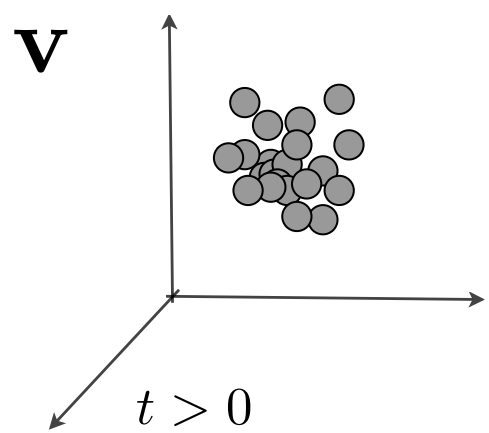


Collisions: Stochastic Differential Equation

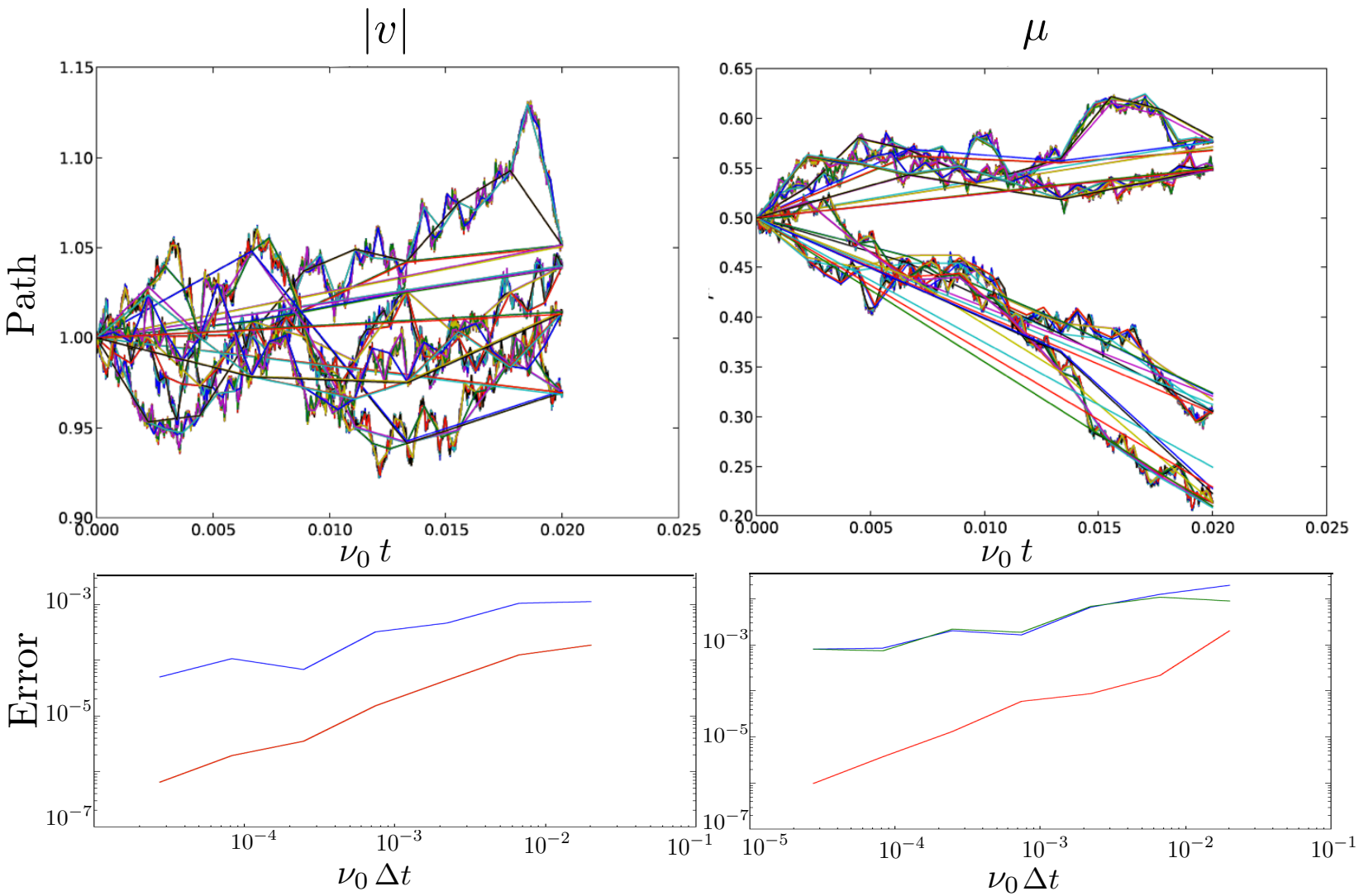
Drag Diffusion

$$\Delta \mathbf{v} = \mathbf{F}_d \Delta t\, \hat{\mathbf{e}}_v + D^a_{v,\mu} \Delta W_{v,\mu} \hat{\mathbf{e}}_{v,\mu} + D^b_\mu \Delta t \hat{\mathbf{e}}_\mu + D^c_{v,\mu} \left(\Delta W^2_{v,\mu} - \Delta t \right) \hat{\mathbf{e}}_{v,\mu} + D^d_\mu \Delta W_{v\star\mu} \hat{\mathbf{e}}_\mu$$





Collisions: Stochastic Differential Equation



1. Cohen, Dimits, Friedman, Caflisch, 2010, *IEEE*, 38, 9, 2396
2. Dimits, Cohen, Caflisch, Rosin, 2011, Work in Progress

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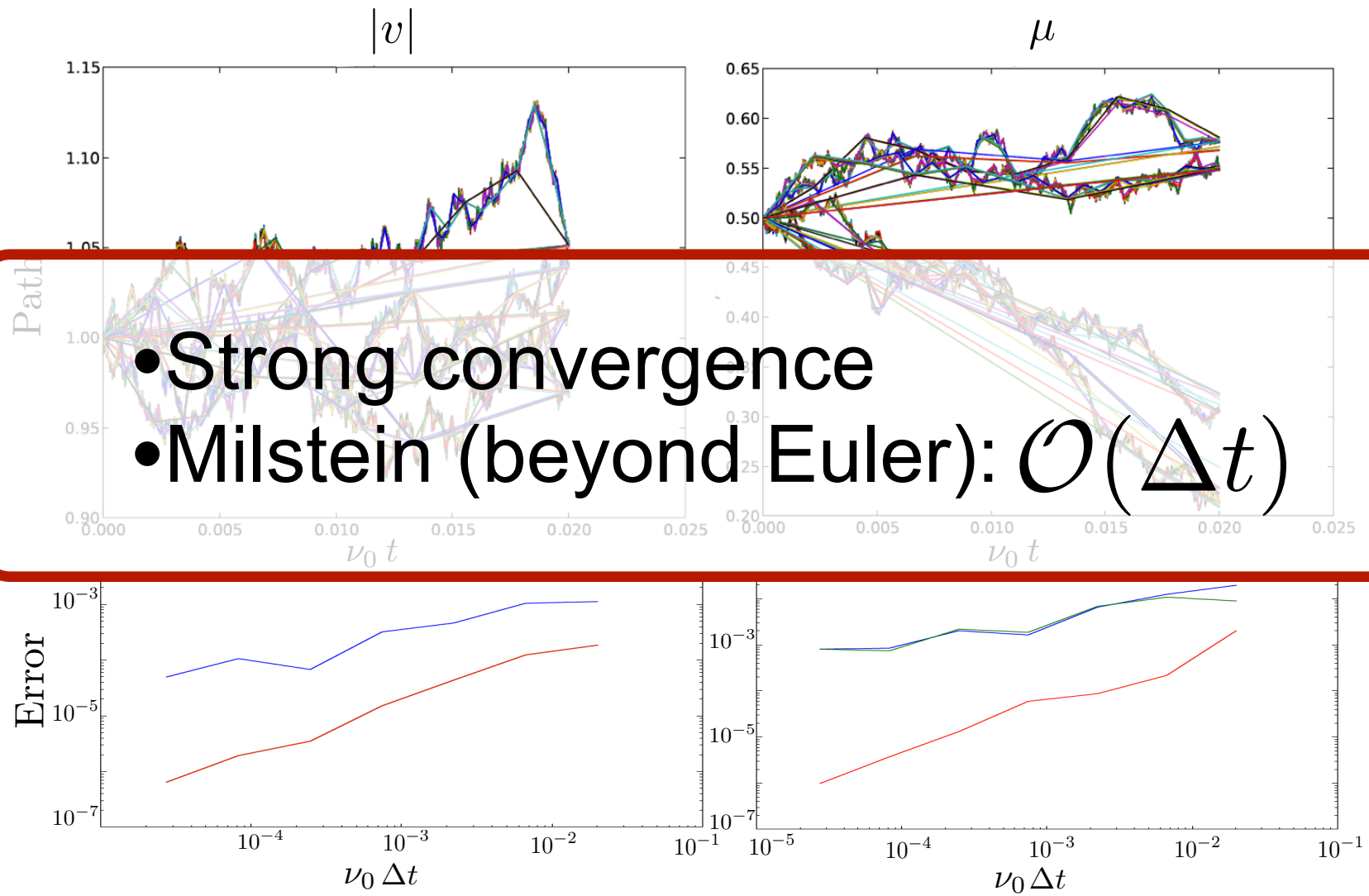
Collisions: Stochastic Differential Equation



- Strong convergence
- Milstein (beyond Euler): $\mathcal{O}(\Delta t)$

1. Cohen, Dimits, Friedman, Caflisch, 2010, *IEEE*, 38, 9, 2396
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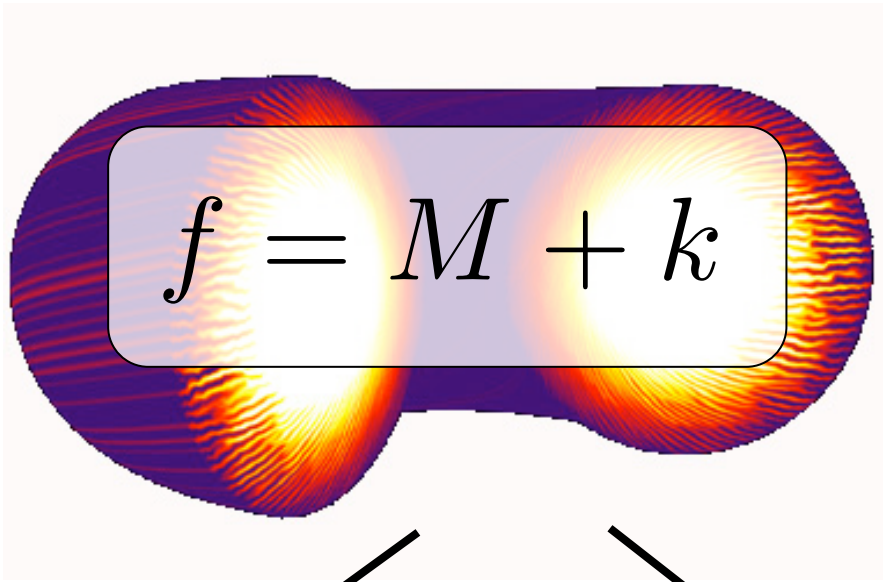
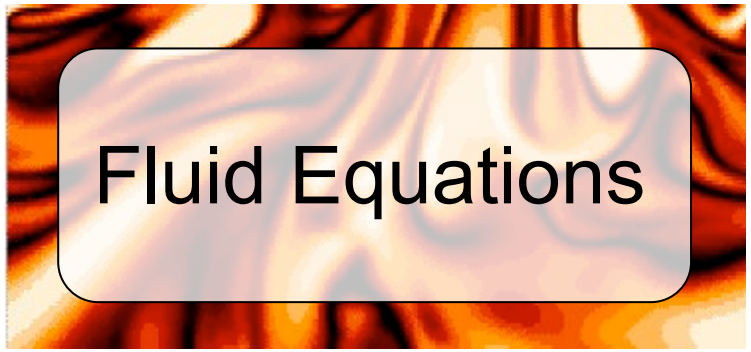
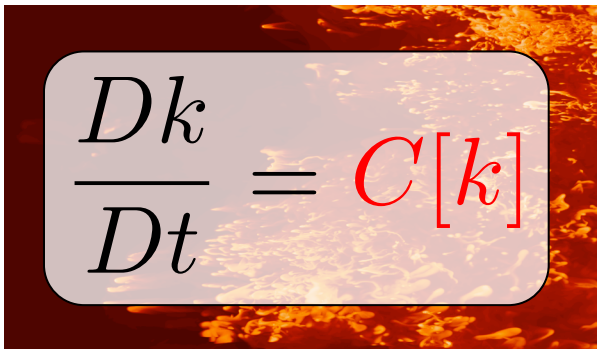
Hybrid Method







Hybrid Method: Formal Decomposition

Fluid Equations $-\overline{C[f,k]} - \overline{S_{\mathrm{de}}} + \overline{S_{\mathrm{th}}}$

$$\frac{Dk}{Dt} = C[f,k] + S_{\mathrm{de}} - S_{\mathrm{th}}$$

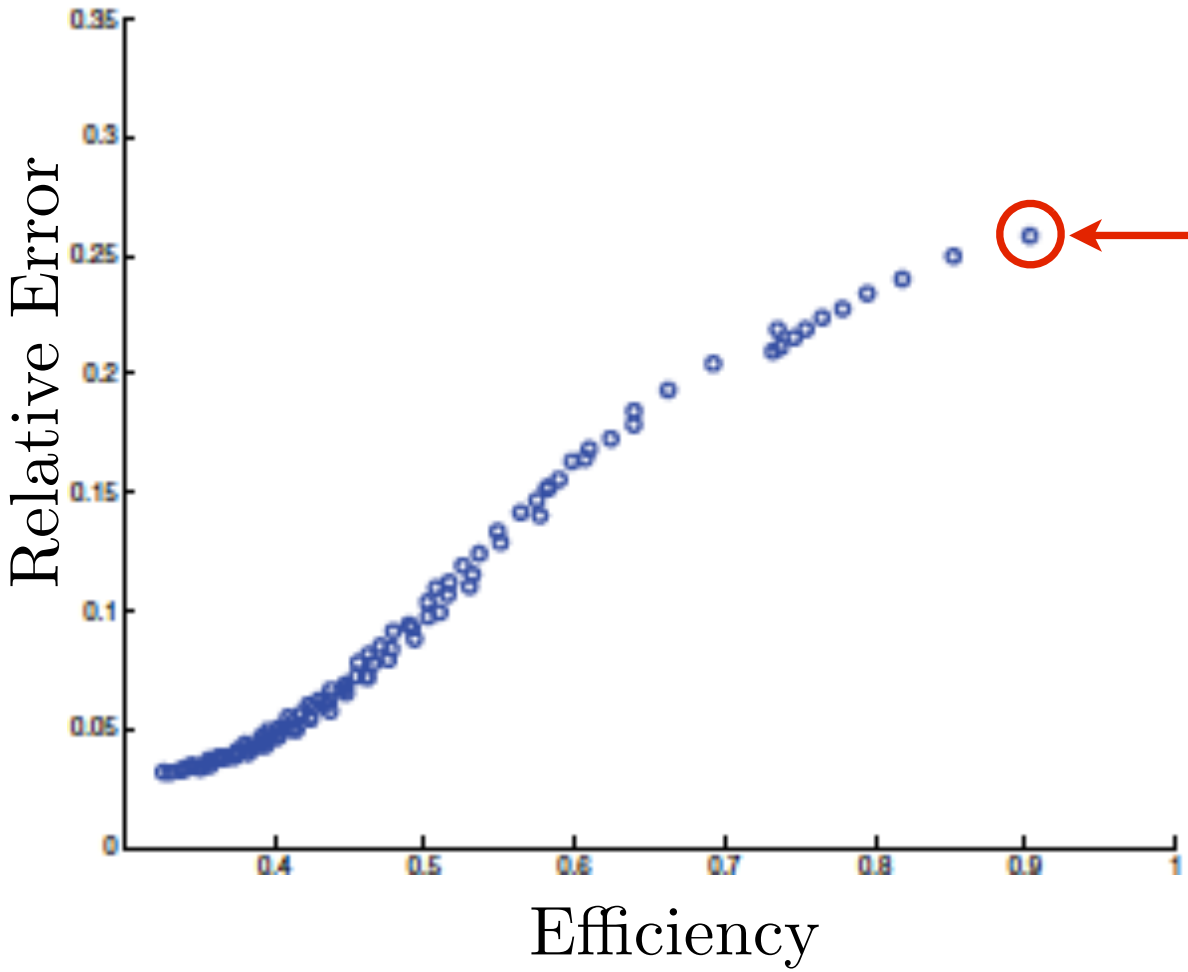
Hybrid Method: Formal Decomposition

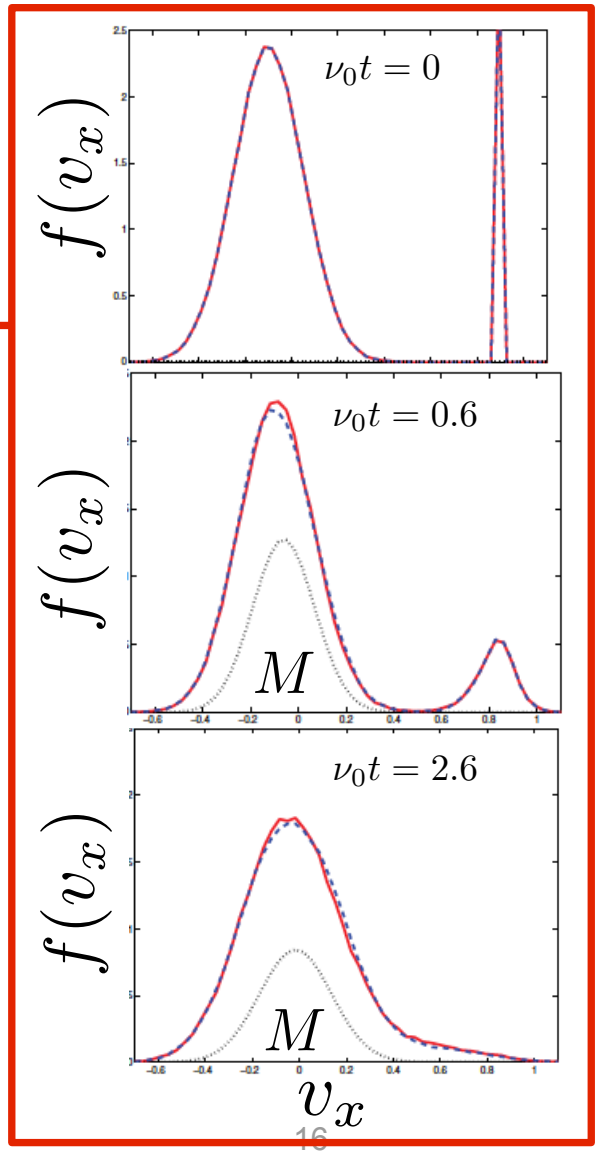
Fluid Equations $-\overline{C[f,k]} - \overline{S_{\mathrm{de}}} + \overline{S_{\mathrm{th}}}$

- Conserves Moments
- Thermalization/Dethermalization freedom
- Accuracy & Efficiency optimization

$$\frac{Dk}{Dt} = C[f,k] + S_{\mathrm{de}} - S_{\mathrm{th}}$$

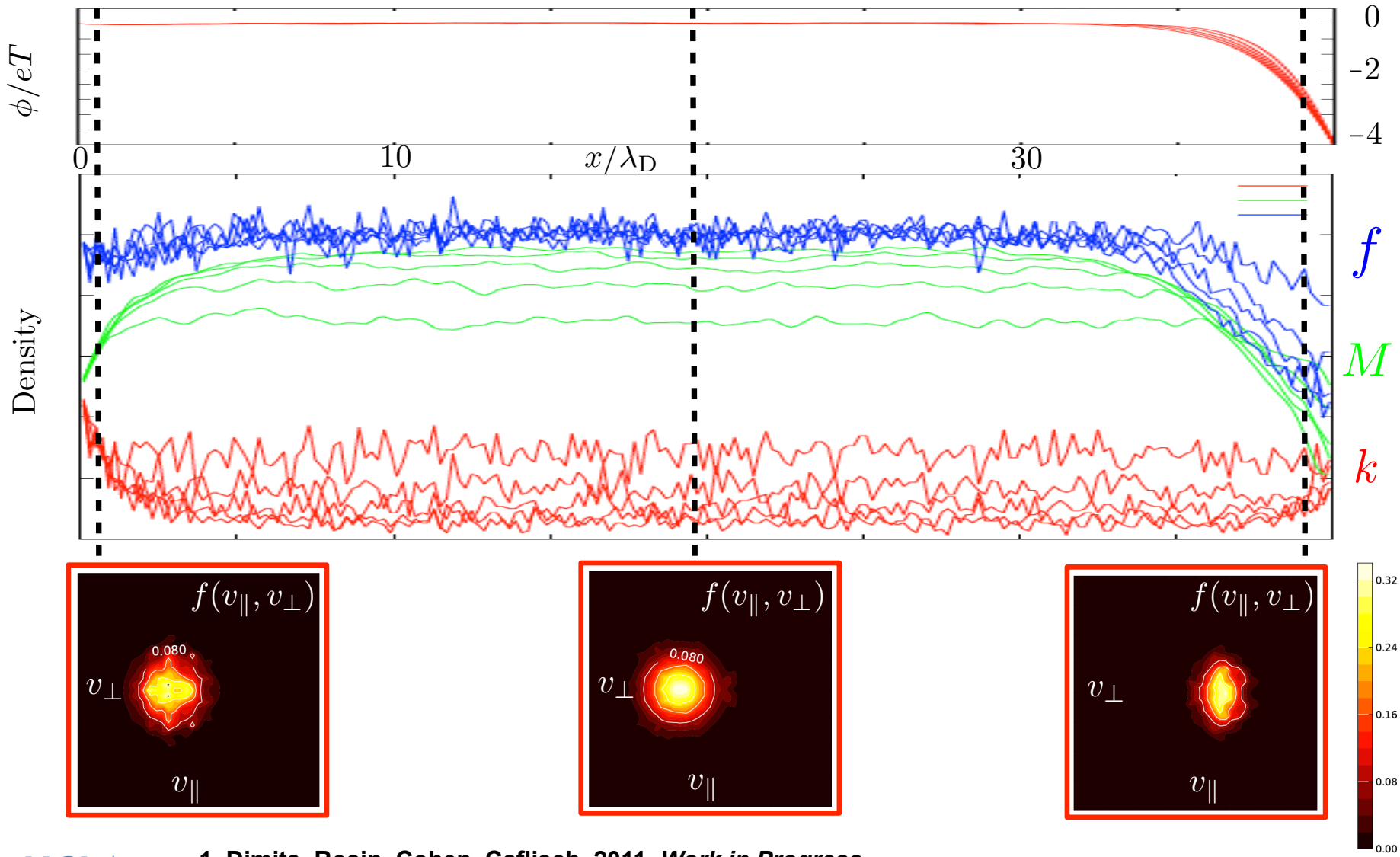
Hybrid Method: Bump-On-Tail Test Case





1. Caflisch, Wang, Dimarco, Cohen, Dimits, 2008, *SIAM,* 7-2, 865

Hybrid Method: Sheath Test Case



1. Dimits, Rosin, Cohen, Caflisch, 2011, *Work in Progress*

Conclusions and Future Work

Conclusions

- Binary algorithms are similar: $\mathcal{O}(\Delta t^{1/2})$
- Strong convergence with Milstein: $\mathcal{O}(\Delta t)$
- Increased efficiency with hybrid algorithm
- Optimization between accuracy and efficiency

Future Work

- Higher-order weak-convergence Langevin equation
- Test Lagrangian and entropy based dethermalization
- Parameter scan for sheath problem
- Implement laser-heating benchmark
- Extend hybrid method to transport equations

Progress at DOE AMP (Oct) and APS DPP (Nov)

Hybrid Method: Inverse-Bremsstrahlung Heating

1. Rosin, Dimits, Cohen, Caflisch, *Work in Progress*