Plasma turbulence simulations and experimental validation on the linear device LAPD

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In collaboration with: T.A. Carter[2], B. Friedman[2], W.M. Nevins[1], P. Popovich[2]

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I. Overview of LAPD experiment and BOUT code
II. Physics model and assumptions
III. Linear analysis
IV. General appearance of computed turbulence
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Presented here material is mostly covered in recent papers :

- P. Popovich, M.V. Umansky, T.A. Carter, and B. Friedman - Phys. Plasmas, 17, 102107 (2010)
- P. Popovich, M.V. Umansky, T.A. Carter, and B. Friedman - Phys. Plasmas, 17, 122312 (2010)
- M.V. Umansky, P. Popovich, T.A. Carter, B. Friedman, and W.M. Nevins - Phys. Plasmas, 18, 055709 (2011)

I. **Overview of LAPD experiment and BOUT code**
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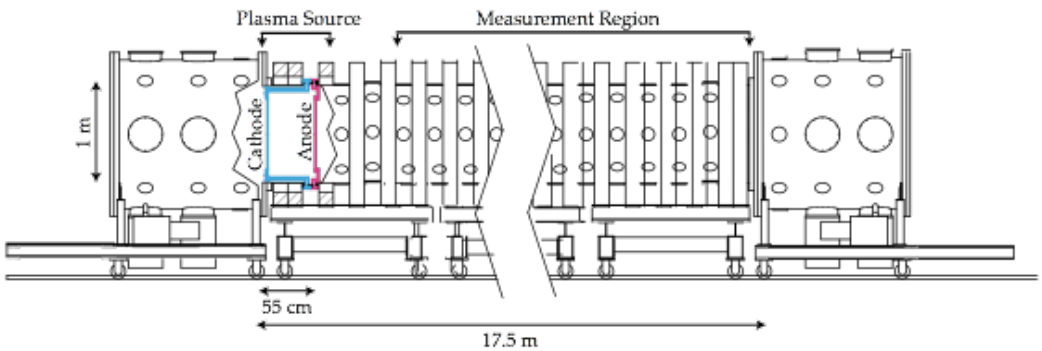
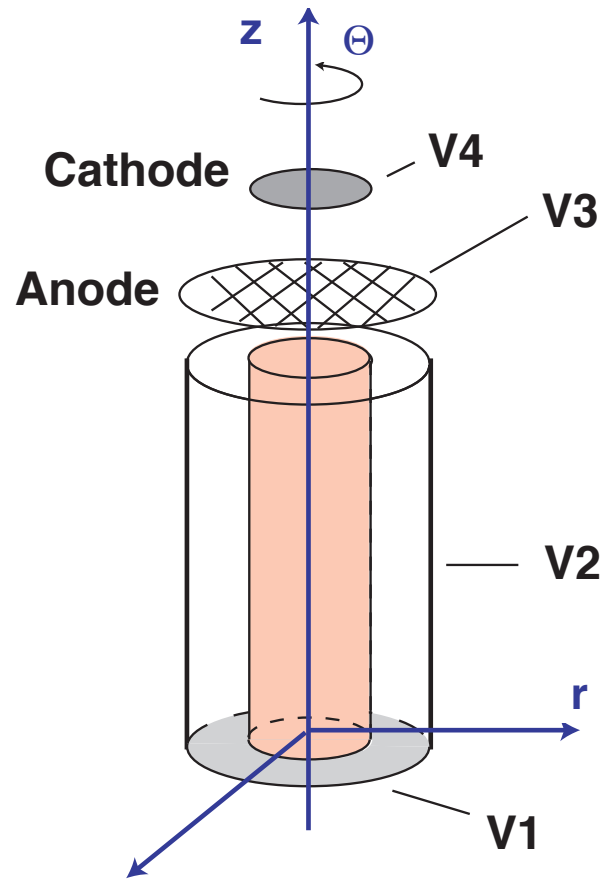
Large Plasma Device (LAPD) is a linear plasma experiment at UCLA





Characteristic parameters of LAPD

N_e	T_e	T_i
2.5e12 cm^{-3}	5 eV	<1 eV

$L_{\|\|}$	a	B
17 m	~0.3 m	~0.1 T

LAPD plasma lies well within domain of validity of collisional fluid plasma model

Parameters in LAPD

Singly-ionized He^+
A=4, Z=1

f_{ci}	380 kHz
ρ_i	0.2 cm
ν_{ei}	7.4e6 1/s
λ_{ei}	10 cm
ω_*	4e4 rad/s

Collisions with neutrals He^0 may be important

$n_N \sim 5\times10^{10} - 5\times10^{11}$ cm^{-3}

$\nu_{in}/\omega_{ci} \sim 2\times10^{-4} - 2\times10^{-3}$

$L_{cx}, L_{iz} \sim 10$-100 m

Compare with tokamak edge plasmas

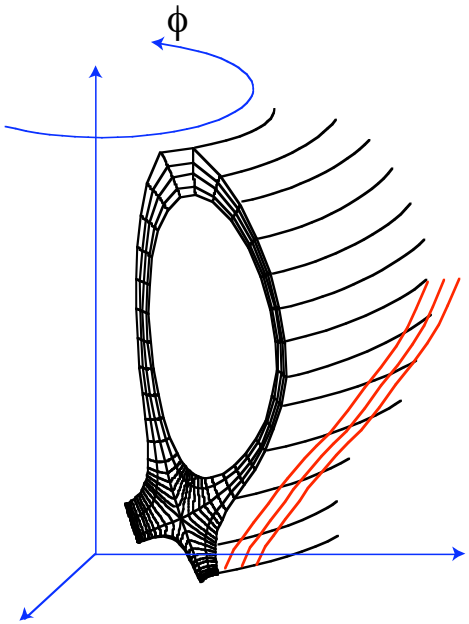
	$k_\perp\rho_i$	$\lambda_{ei}/L_{\parallel}$	ω/ν_{ei}
DIII-D	1.0	1.0	1.0
C-Mod	0.1	0.1	0.1
NSTX	1.0	0.1	0.1
LAPD	0.1	0.01	0.01

LAPD can be used as a test-bed for collisional plasma models validation

BOUT (BOUndary Turbulence) was originally developed at LLNL in late 1990s for modeling tokamak edge turbulence*

- Time integration of dynamic equations for collisional plasma fluid (Braginskii)
- Spatial discretization on 3D mesh in real geometry
- Time integration by implicit ODE solver PVODE
- Parallel implementation with MPI
- BOUT++ is a newer improved version#
- Results reported here produced with older BOUT; now using BOUT++ for LAPD⌘

Field following grid



X.Q. Xu and R.H. Cohen, Contrib. Plasma Phys. 38, 158 (1998)

Dudson, Umansky, Xu et al., Comp. Phys. Comm. 180, 1467 (2009)

⌘*B. Friedman et al., APS DPP 2010 poster TP9.00014*

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For LAPD calculations 3-field fluid model for cylindrical geometry is implemented in BOUT

$$(\partial_t + V_e \bullet \nabla) n = 0$$

$$n m_e (\partial_t + V_e \bullet \nabla) V_e = -\nabla p_e - ne(E + \frac{1}{c} V_e \times B) - n m_e \nu_{ei}(V_e - V_i) - n m_e \nu_{en} V_e$$

$$n m_i (\partial_t + V_i \bullet \nabla) V_i = ne(E + \frac{1}{c} V_i \times B) - n m_i \nu_{in} V_i$$

$$\nabla \bullet J = 0, \quad J = ne(V_{e\|} - V_{i\|}) + ne(V_{e\perp} - V_{i\perp})$$

Conservation of
- ✓ **plasma density**
- ✓ **momentum**
- ✓ **electric charge**

<u>Model assumptions</u>

$B=B_z$=const
T_e=const
T_i=0
$k_\perp \rho_i \ll 1$
$\lambda_{ei}/L_\| \ll 1$
$\omega/\nu_{ei} \ll 1$

Electrostatic Model

Equations implemented in the code

$$\partial_t N = -V_E \bullet \nabla N - V_{\|e} N$$

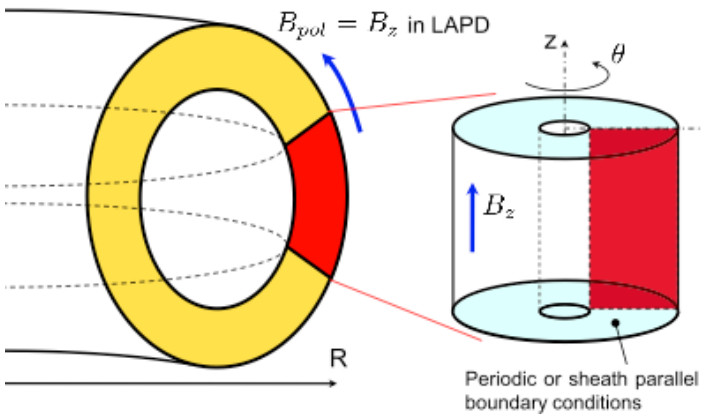
$$\partial_t V_{\|e} = -V_E \bullet \nabla V_{\|e} - \mu \frac{T_{e0}}{N_0} \nabla_\| N + \mu \nabla_\| \phi - \nu_e V_{\|e}$$

$$\partial_t \varpi = -V_E \bullet \nabla \varpi - \nabla_\| (N V_{\|e}) + b \times \nabla N \bullet \nabla (V_E^2/2) - \nu_{in} \varpi + \mu \nabla_\perp^2 \varpi$$

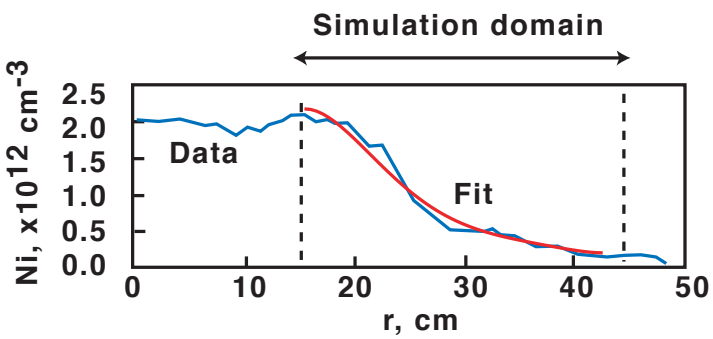
$$\varpi \equiv \nabla_\perp \bullet (N \nabla_\perp \phi)$$

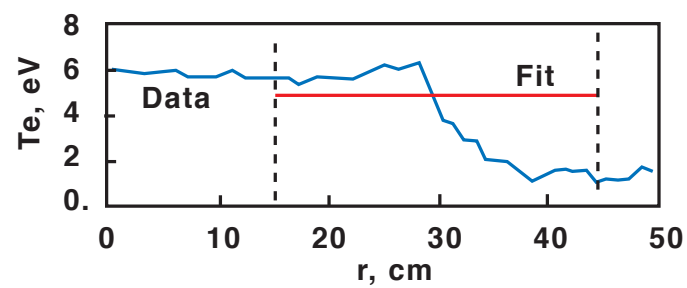
Geometry, plasma profiles, and boundary conditions are set up to match LAPD

Geometry is cylindrical annulus



N_{i0} (r) is fitted to experimental profile
T_e=5 eV =const
T_i=0





Azimuthal periodic (domain $\pi/4$, π, 2π)
Axial B.C. – periodic

Radial B.C. – Γ=0 at r_{min} & r_{max}

$$r_{\min} \rightarrow \partial_r \langle \phi \rangle_\theta = \tilde{\phi} = \partial_r \tilde{N} = 0$$
$$r_{\max} \rightarrow \langle \phi \rangle_\theta = \tilde{\phi} = \partial_r \tilde{N} = 0$$

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Linear general dispersion relation has three instability branches – drift, interchange, Kelvin-Helmholtz

- **Linearize and assume the form**

$$f(r)\exp(im\theta + in_z z/L_{\parallel} - i\omega t)$$

- **Obtain an eigenvalue problem**

$$C_2(r,\omega)f'' + C_1(r,\omega)f' + C_0(r,\omega)f = 0$$

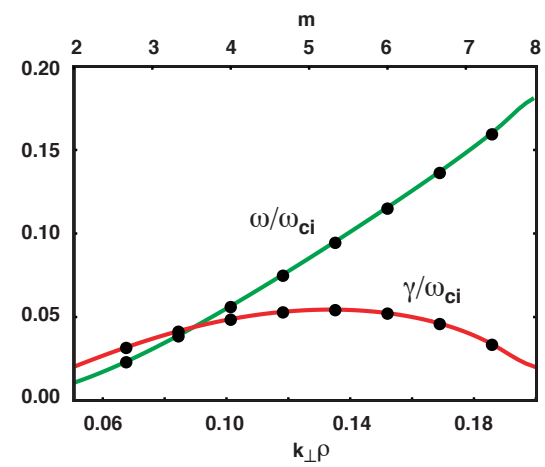
- **One branch is resistive drift wave (DW)**

- **Due to ϕ_0 there are also other branches:**
  - **Rotational Interchange (IC)**
  - **Kelvin-Helmholtz (KH)**

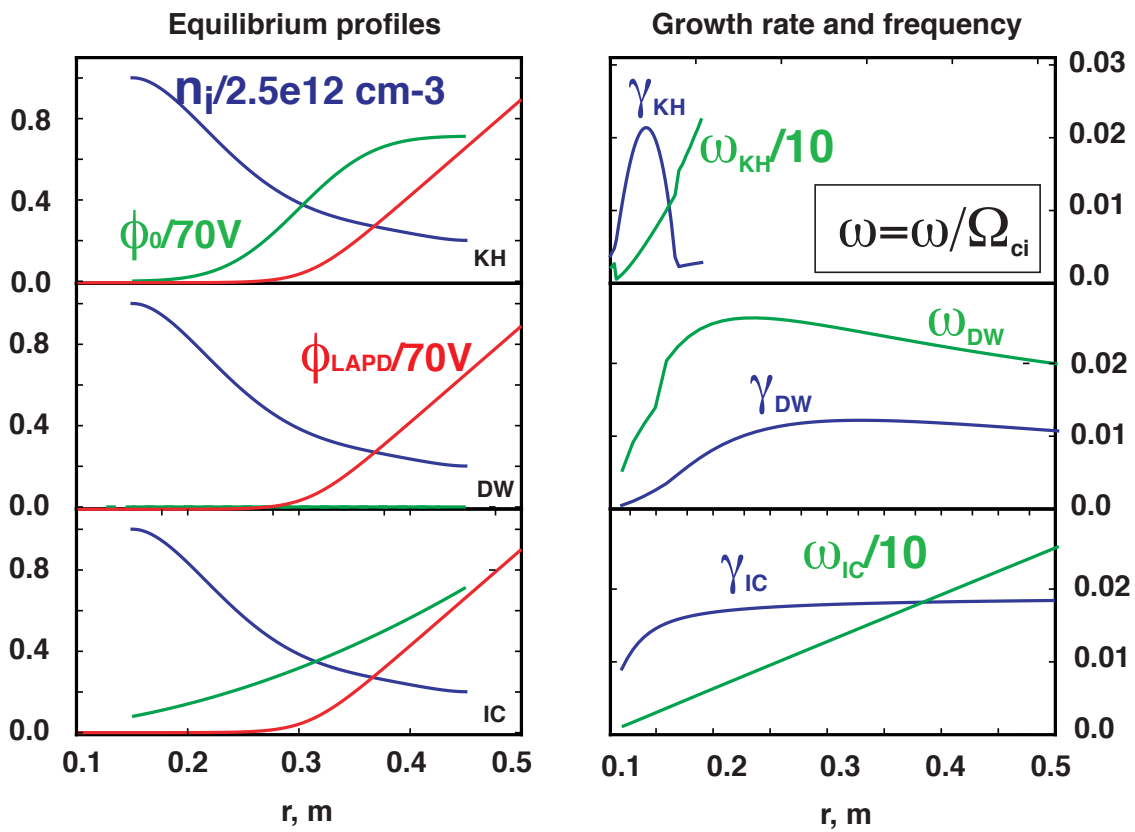
- **Detailed BOUT verification has been done on this linear dispersion relation**

Linear verification example

KH mode in LAPD: BOUT (dots) vs. analytic



For LAPD parameters all three modes have comparable growth rates at similar azimuthal numbers ~10



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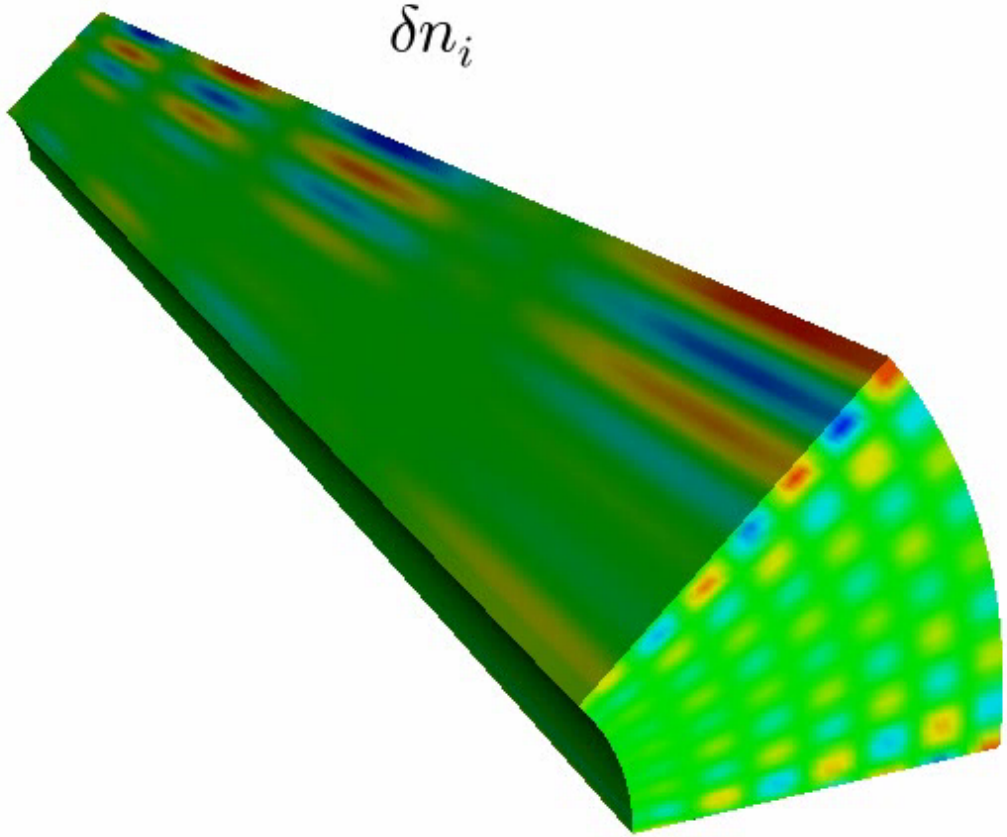
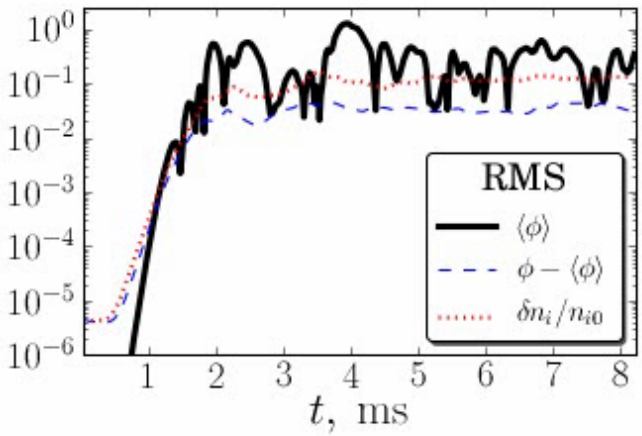
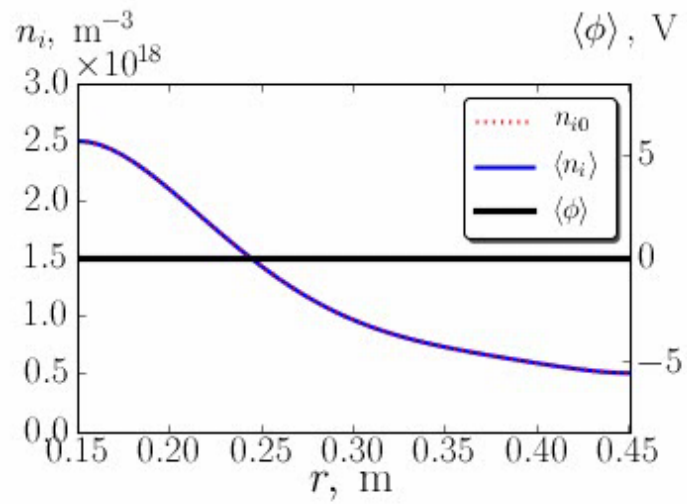
Time-integration of equations results in linear growth and saturated turbulence

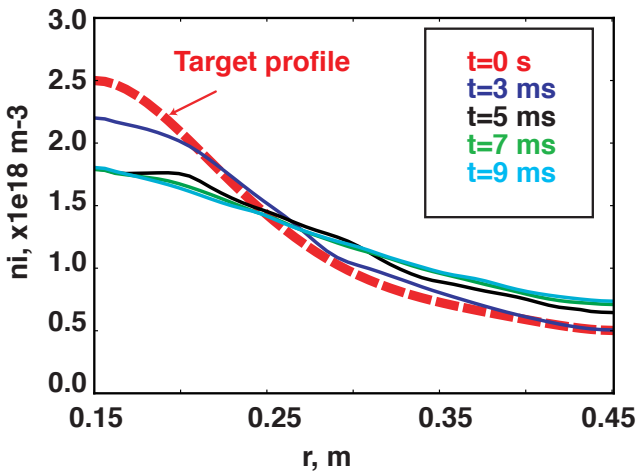




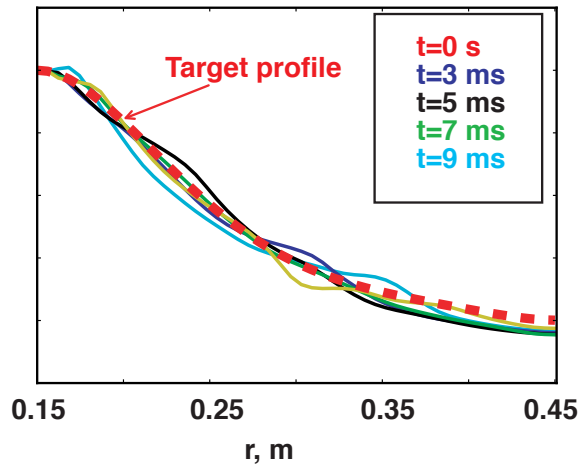


Density profile control is realized either by suppressing m=0, or by using a source term

Without control, $N_i(r)$ profile drifts away from the target $N_{i0}(r)$



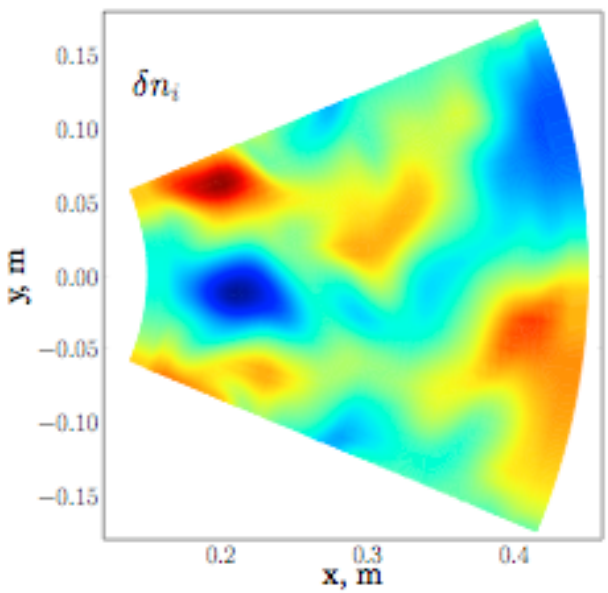
With S(r) term, $N_i(r)$ oscillates around the target $N_{i0}(r)$



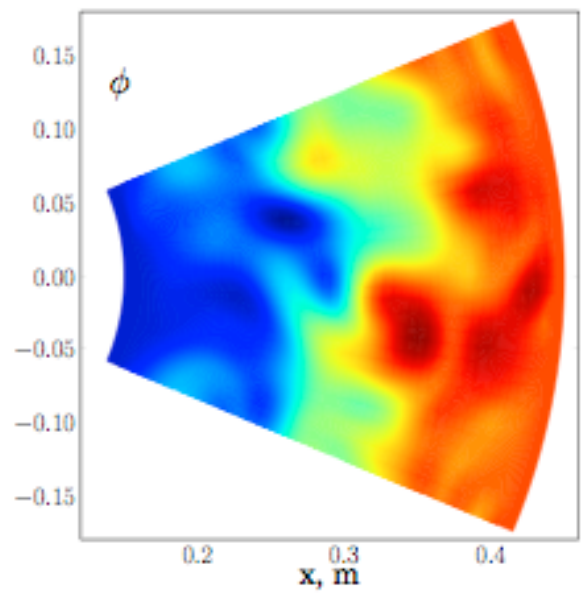
- **First suppress m=0 component, then calculate Γ and S(r)=div(Γ)**
- **Now run using S(r) and keeping m=0**
- **Both profile control methods produce similar results**
- **More "physical" source model can be constructed using ionization, and including parallel flow and end-plate boundary conditions**

Turbulence leads to self-generated zonal flow

Density perturbations
m=0 suppressed
<N(r)>=N_0(r) fixed



Potential perturbations
m=0 not constrained =>
allows for zonal flow



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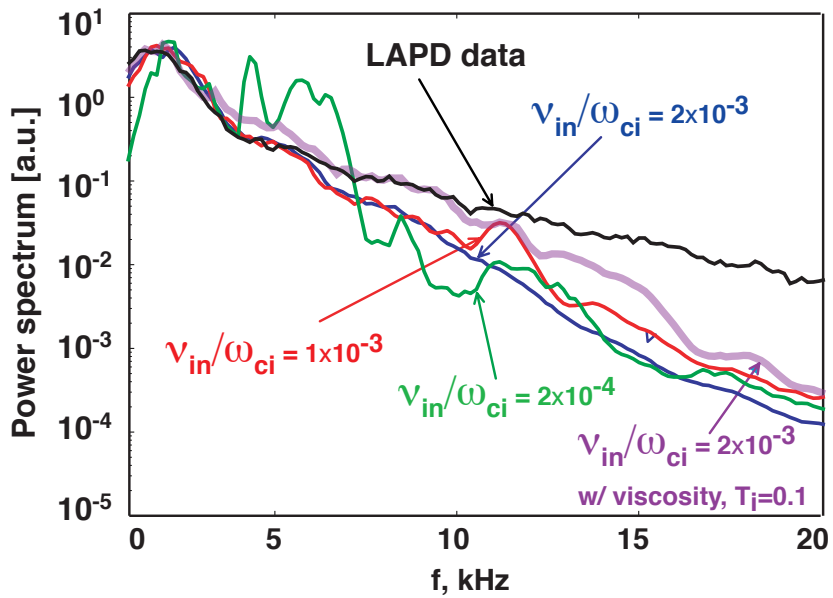
Temporal spectra display qualitative and semi-quantitative similarity to LAPD data

Power spectrum for several values of ν_{in} and μ_{ii}



Using ion viscosity for T_i=0.1 eV yields close agreement for f<10 kHz

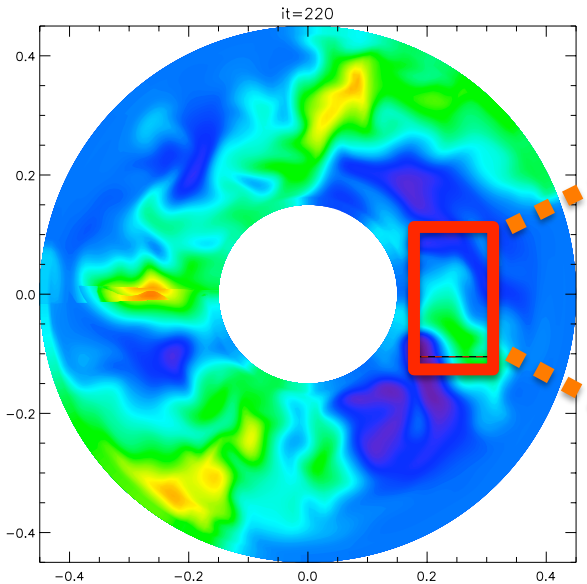
- **High-frequency part shows exponential decay like in LAPD**

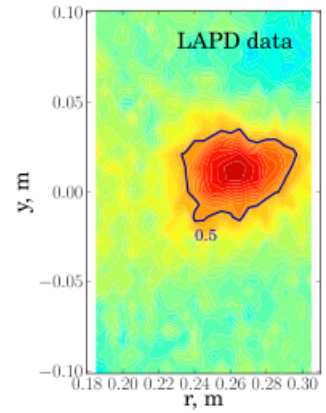
- **Neutrals are important:**
  - ✓ **too little => coherent features not seen in LAPD**
  - ✓ **too much => turbulence is damped**
  - ✓ **using experimental n_N range seems to work best**

- **Ion viscosity is taken at T_i=0.1 eV**
  - **T_i is not well known, < 1 eV**
  - **At T_i ~0.1 eV, ω_{ci}~ν_{ii} ions may be not magnetized**

Spatial correlation length is 2-3 times larger than in LAPD data

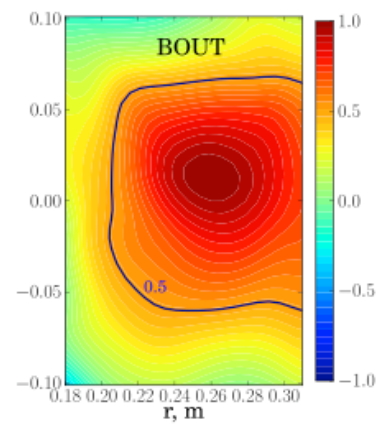
Cross-section Z=const





Experiment

L_{corr}~2.5 cm



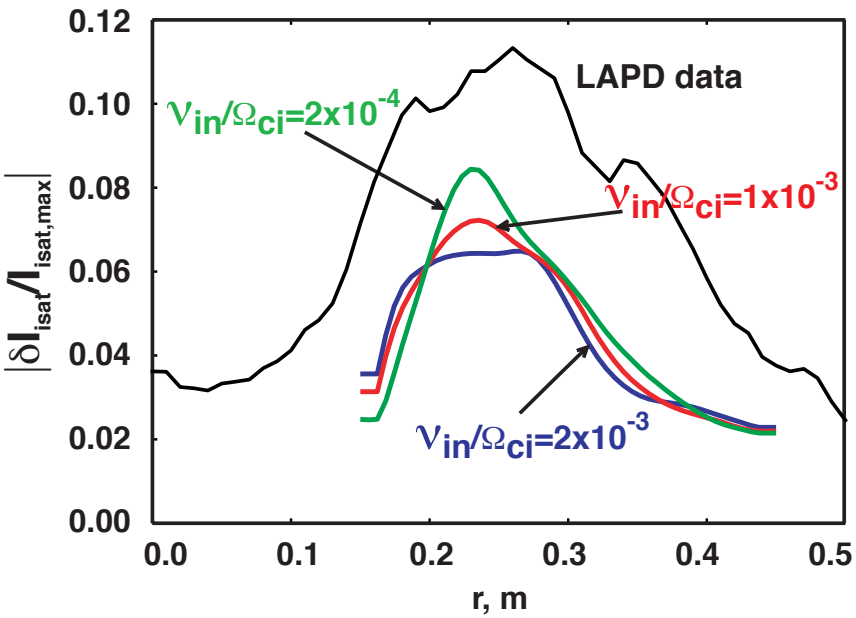
Simulation

L_{corr}~5-7 cm

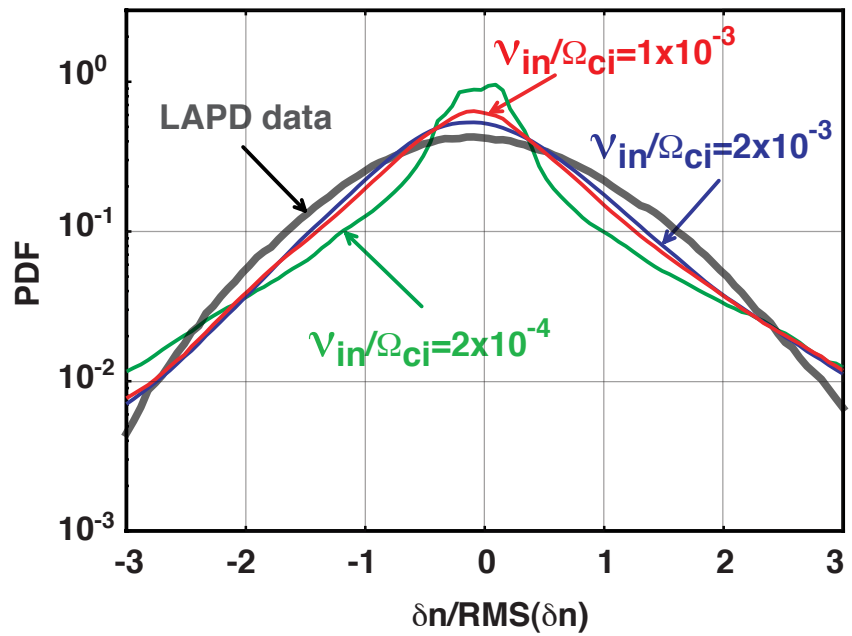
The system has large range of spatial scales: from ρ_i~0.1 cm to a~10 cm; simulation agrees with LAPD data by order of magnitude

Calculated fluctuations amplitude is within factor ~2 from LAPD data, qualitatively in reasonable agreement

Spatial distribution of RMS amplitude



PDF of δn/n is in semi-quantitative agreement with experimental data

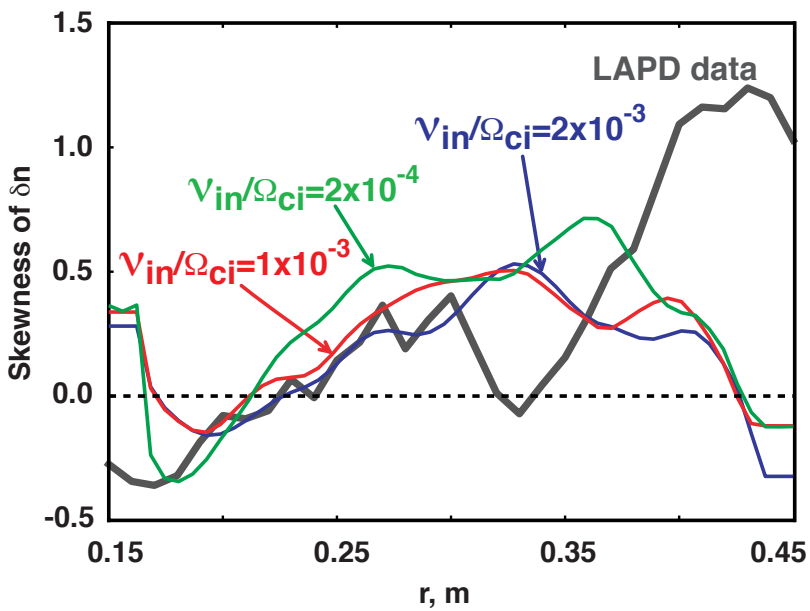


✓Synthetic diagnostics are used

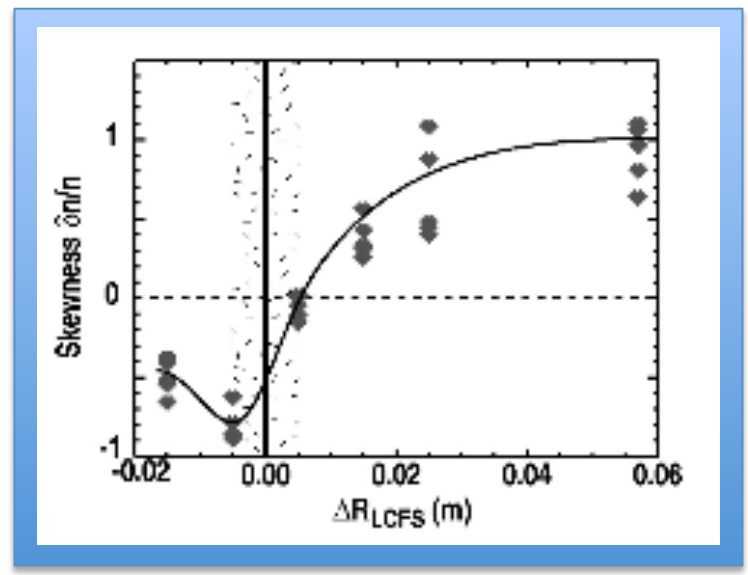
✓If neutral density is taken too low => PDF shape becomes different from expt.

Radial profile of δn/n PDF skewness bears similarity to tokamak edge data

In LAPD data and BOUT results the skewness grows radially



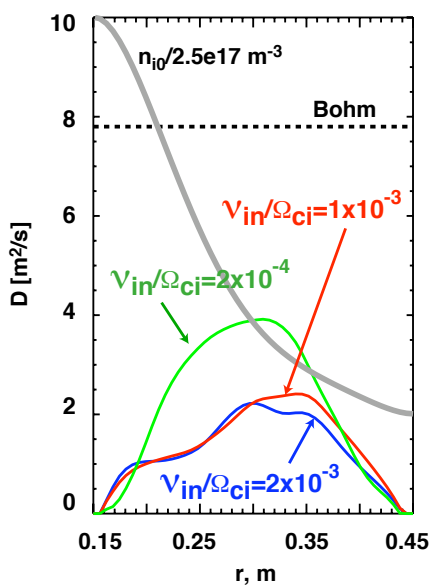
In tokamak edge the skewness grows radially from negative to positive, ~1



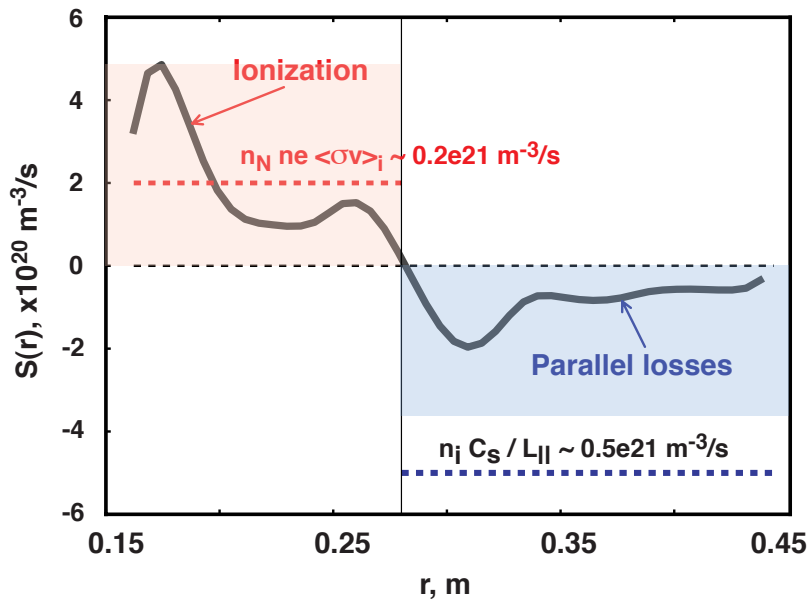
From J.A. Boedo et al., Phys. Plasmas, Vol. 10, No. 5, p. 1670, 2003

Effective plasma source inferred from calculated radial particle flux is consistent with data

Turbulent particle flux follows density gradient



Effective source S=div•Γ_{turb} consistent with LAPD data



For non-rotating LAPD plasmas, particle transport was previously found to be consistent with Bohm

J.E. Maggs, T.A. Carter, and R.J. Taylor, Phys. Plasmas, 14, 052507 (2007).

Calculated electric field <E_r> evolution is consistent with azimuthal momentum balance

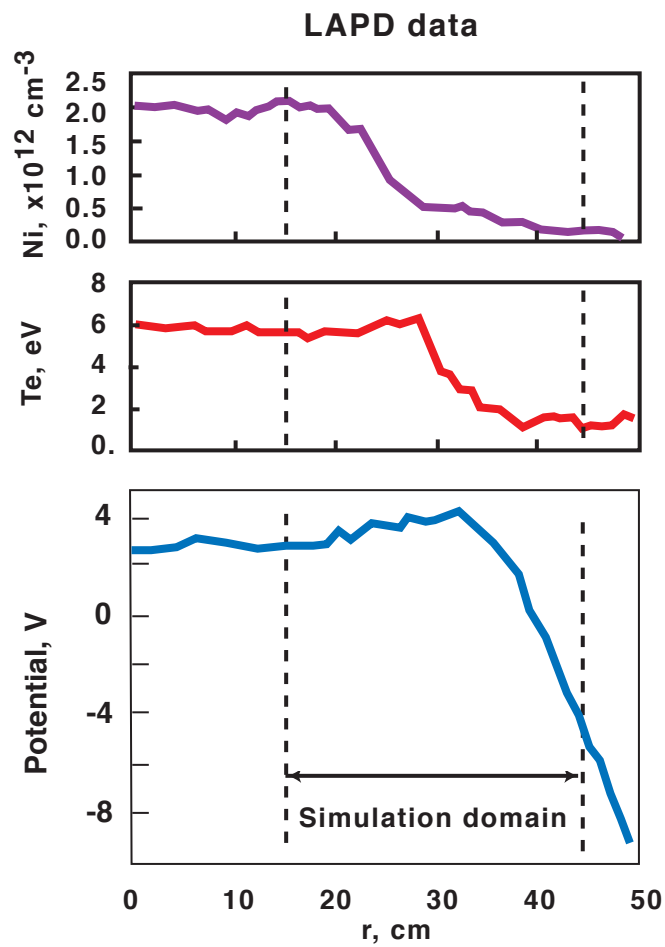
dE_r/dt | Neutral drag | Reynolds stress | Viscous damping

$$\frac{\partial}{\partial t}\langle NV_\theta\rangle = -\nu_{in}\langle NV_\theta\rangle + \langle V_r\varpi\rangle - \mu\frac{\partial}{\partial r}\langle\varpi\rangle$$

$$\frac{Neutral\ drag}{Viscous\ damping} \sim \frac{\nu_{in}/\nu_{ii}}{(\rho_{ci}/a)^2} \sim 10$$

BOUT indicates δ<ϕ> ~ 1-3 V

In experiment E_r is probably set by combination of turbulence effects and boundary conditions



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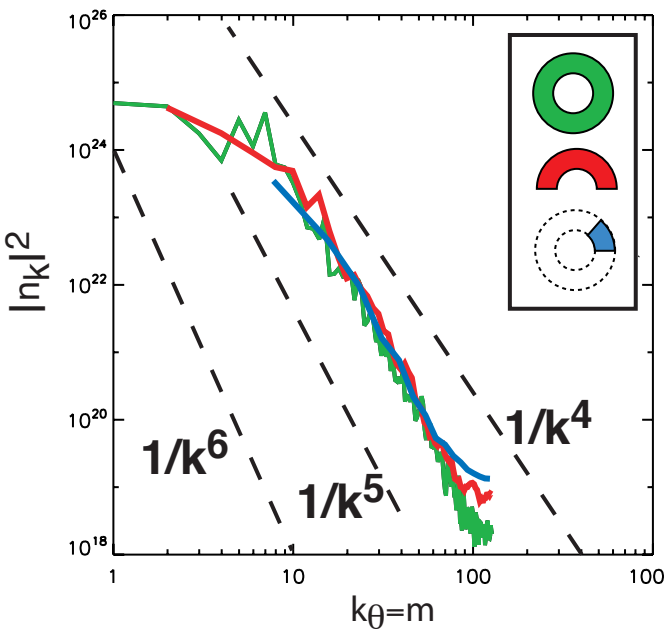
Spectrum in azimuthal wavenumber k_θ indicates presence of direct and inverse cascades

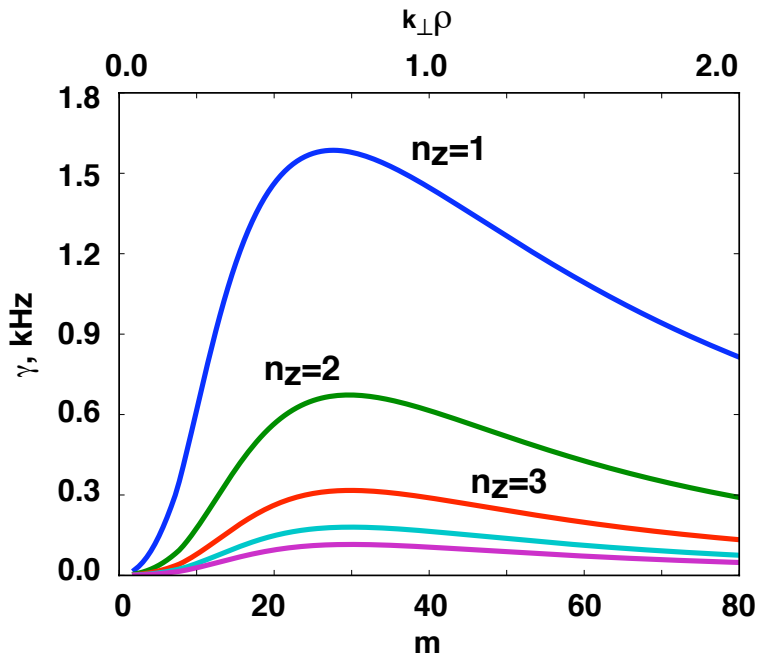
Turbulence spectrum

- ✓Asymptotic scaling $n_k \sim 1/k^{2\text{-}3}$
- ✓Breakpoint at m~20



Spectrum of linear drift instability

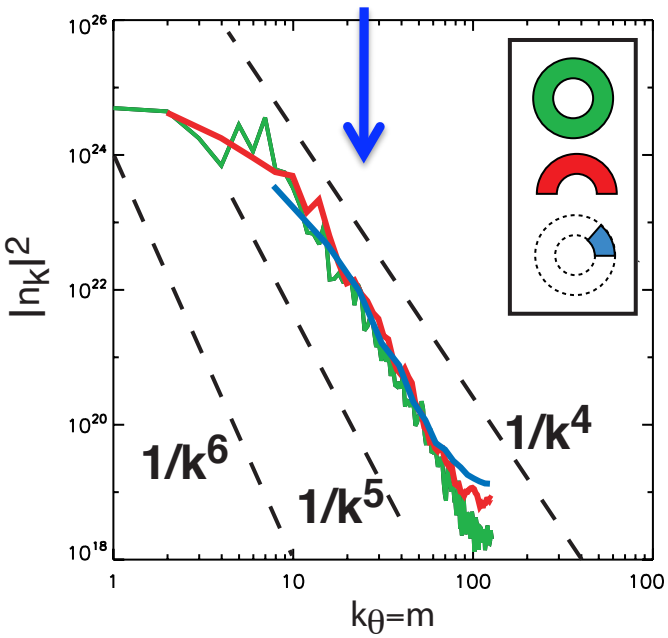
- ✓Peaking at m~25 consistent with breakpoint in turbulence



Spectrum in azimuthal wavenumber k_θ indicates presence of direct and inverse cascades

Turbulence spectrum

- ✓Asymptotic scaling $n_k \sim 1/k^{2\text{-}3}$
- ✓Breakpoint at m~20



Spectrum of linear drift instability

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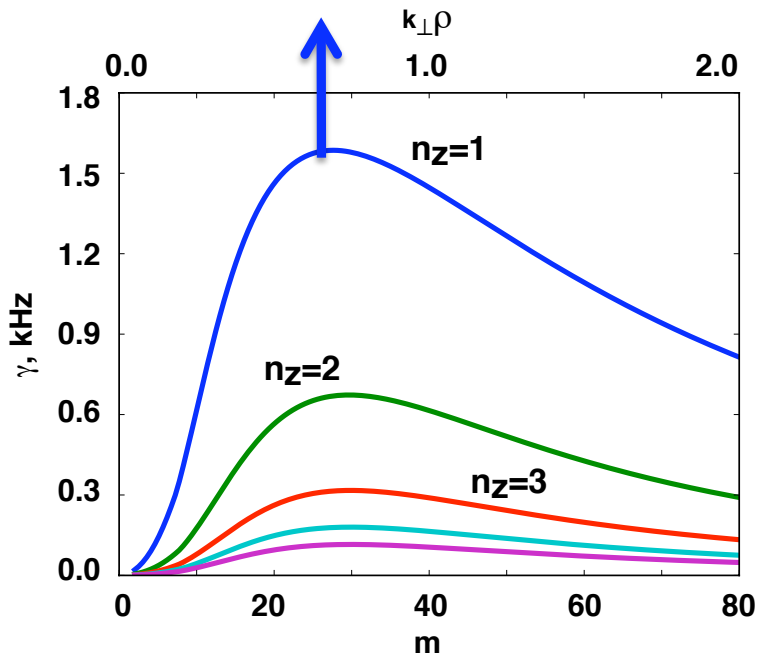
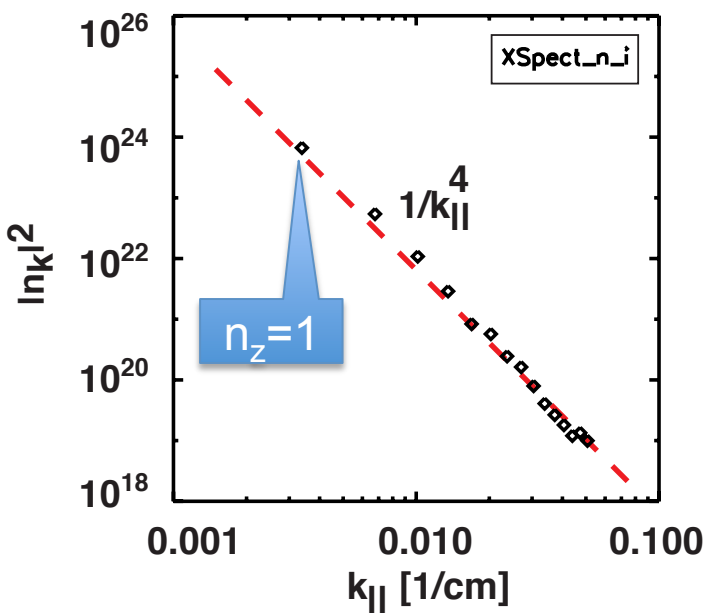
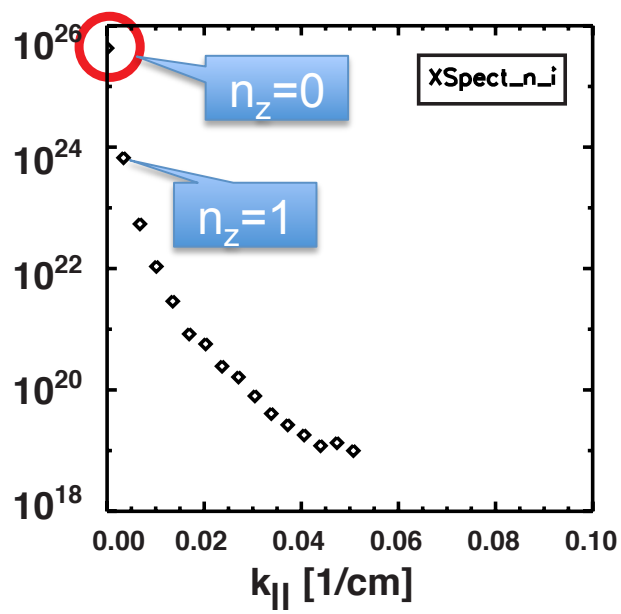


Turbulence spectrum in axial wavenumber $k_{||}$ indicates cascade to higher $k_{||}$ and coupling to low $k_{||}$

- Asymptotic scaling vs. $k_{||}$: $|n_k|^2 \sim 1/k^4$
- Linear drift instability peaks at n_z=1
- However n_z=0 mode strongly dominates in saturated turbulence
- Drift instability =>Reynolds stress=>E_r=>KH instability?

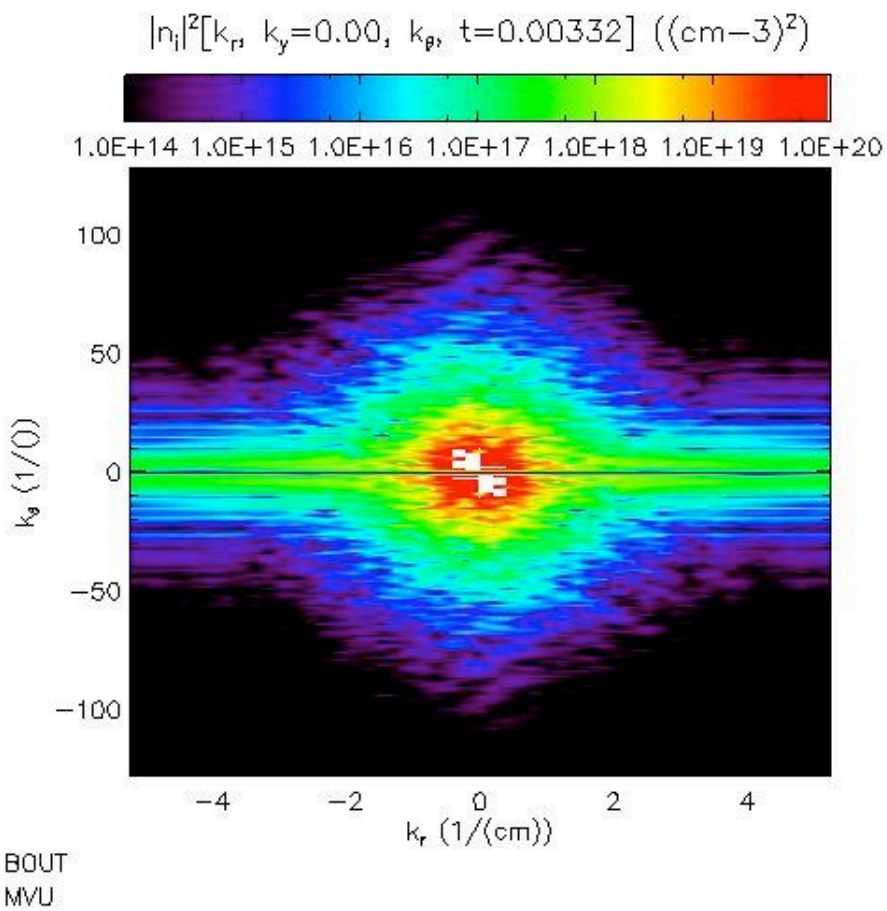




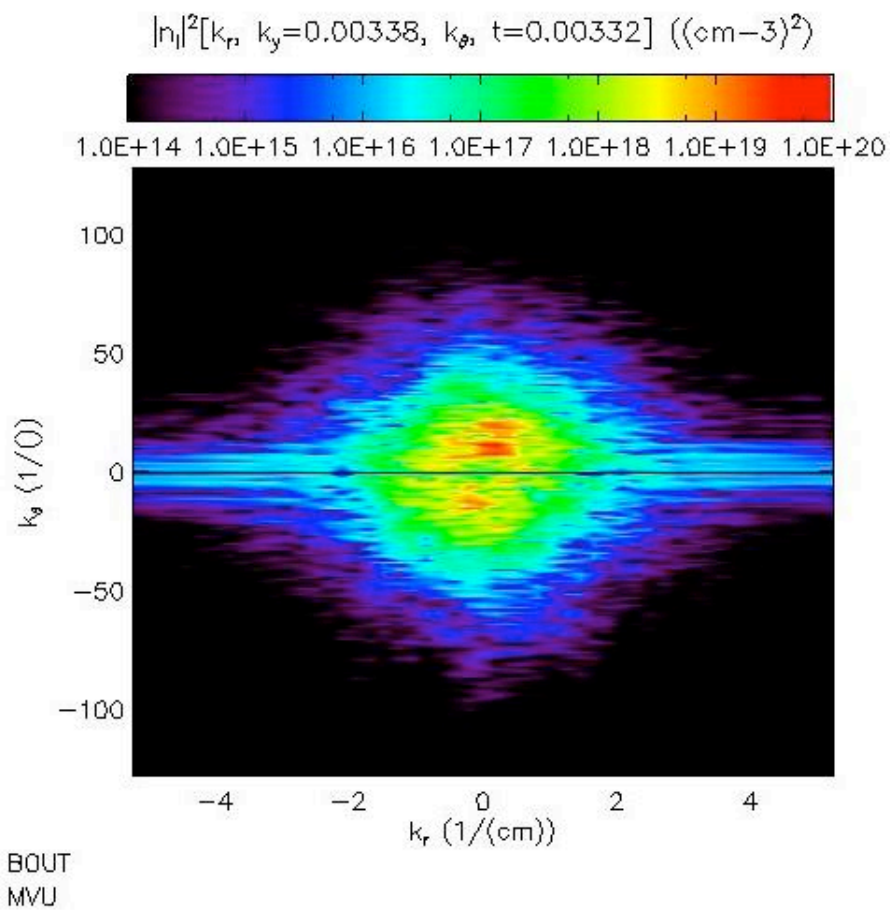
Sequence of events shows that n_z=1 mode emerges first, it apparently drives n_z=0 mode

$|n_k|^2$ vs. k_r, k_θ

n_z=0:



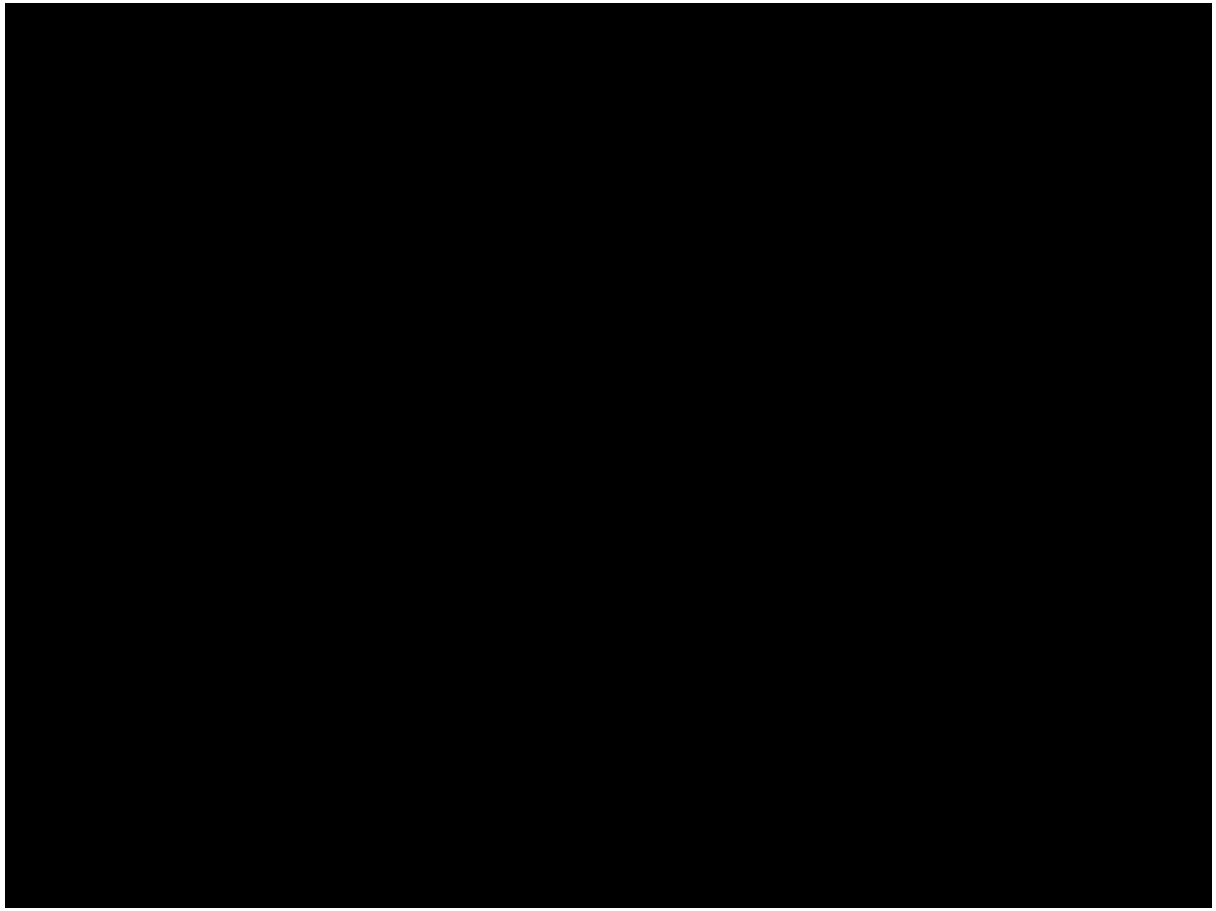
n_z=1



Sequence of events shows that n_z=1 mode emerges first, it apparently drives n_z=0 mode

$|n_k|^2$ vs. k_r, k_θ

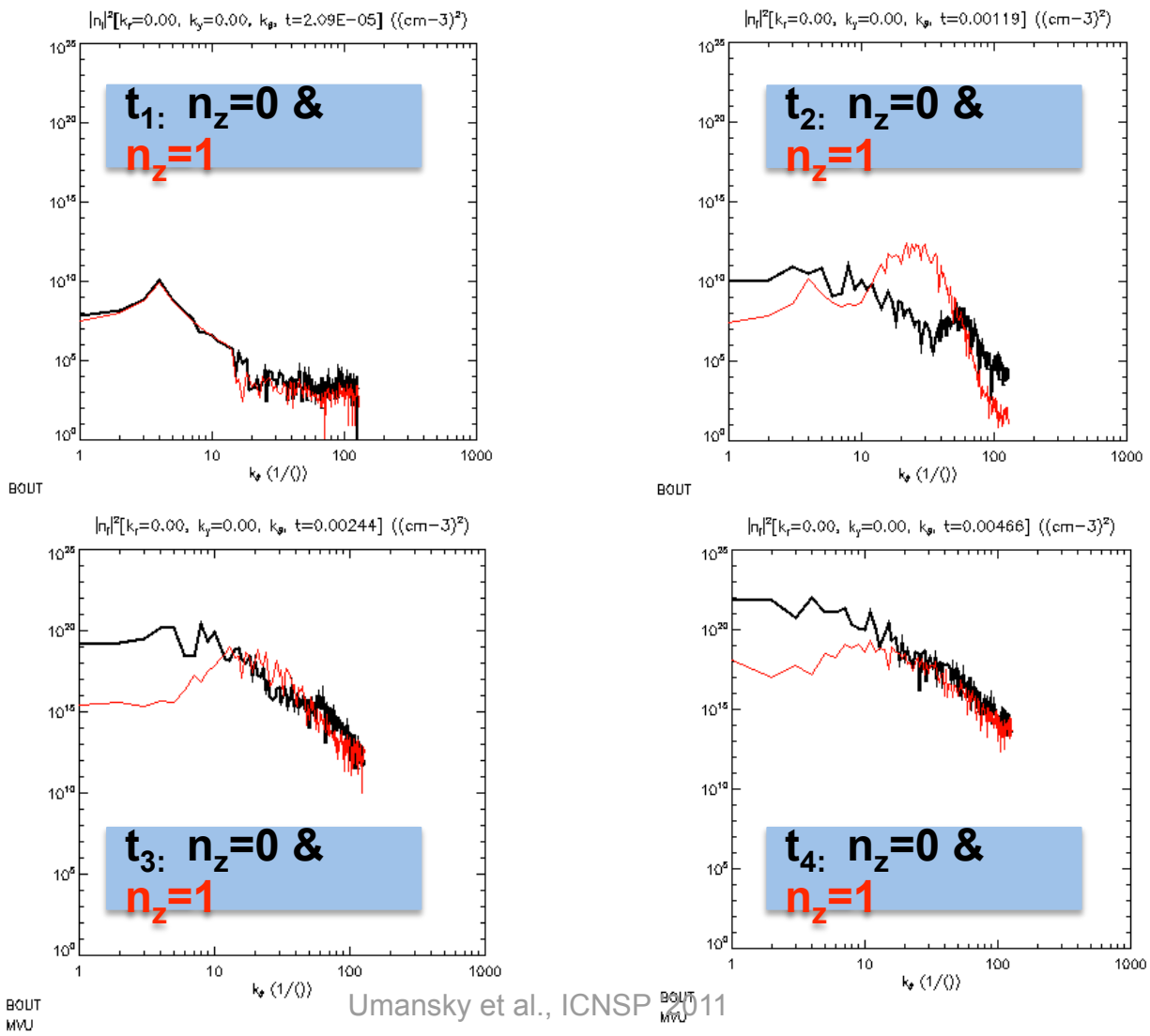
n_z=0: n_z=1



Sequence of events shows that n_z=1 mode emerges first, it apparently drives n_z=0 mode

Plotted

$|n_k|^2$ vs. m



Bicoherence analysis indicates coupling of two drift modes (n_z=±1,m=±25) and a flute-like mode (n_z=0,m~0)

Bicoherence is a measure of strength of coupling between considered modes

Considering three modes (n_z,m_θ)

W_1=(1,25), W_2=(-1,-24), W_3=(0,1)

For three Fourier modes that satisfy sum rules

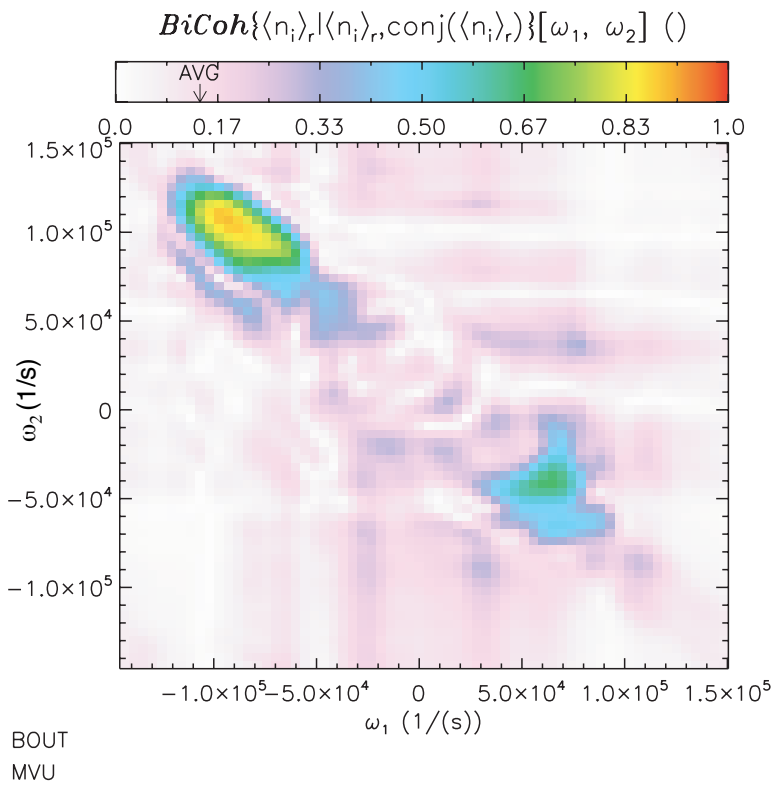
$$\omega_1 + \omega_2 = \omega_3 \qquad k_1 + k_2 = k_3$$

bispectrum is defined as ensemble average

$$S(\omega_1,\omega_2) = \left\langle \hat{Y}^*(\omega_3,k_3)\,\hat{Y}(\omega_1,k_1)\,\hat{Y}(\omega_2,k_2) \right\rangle$$

bicoherence is defined as

$$b(\omega_1,\omega_2) = \frac{|\,S(\omega_1,\omega_2)\,|}{\sqrt{|\,\hat{Y}^*(\omega_3,k_3)\,|^2\,|\,\hat{Y}(\omega_1,k_1)\hat{Y}(\omega_2,k_2)\,|^2}}$$



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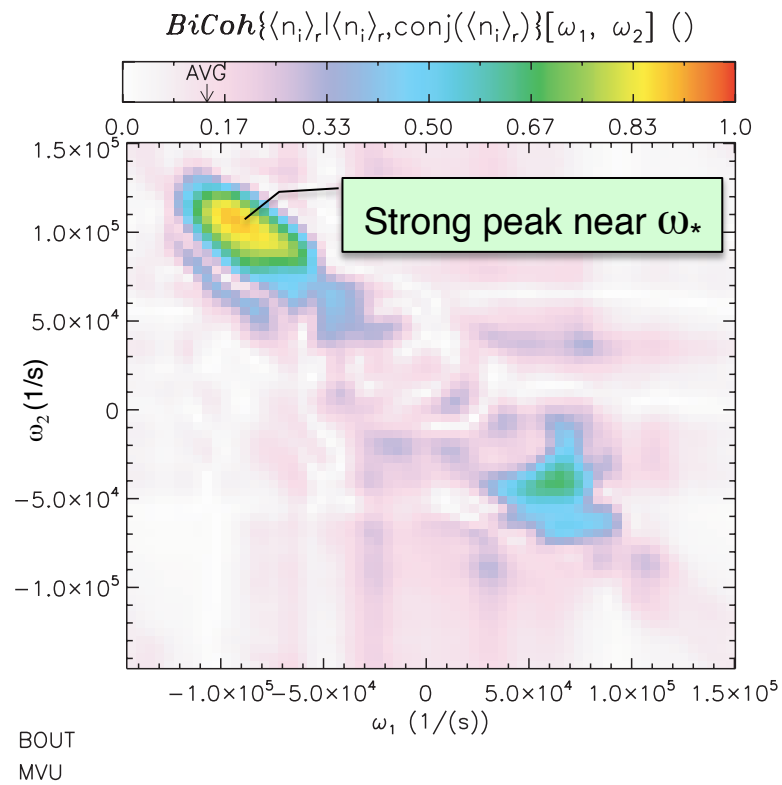
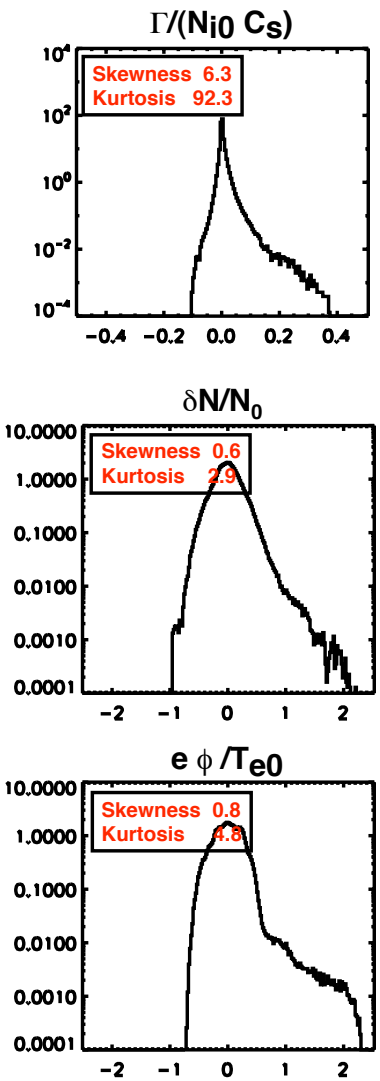


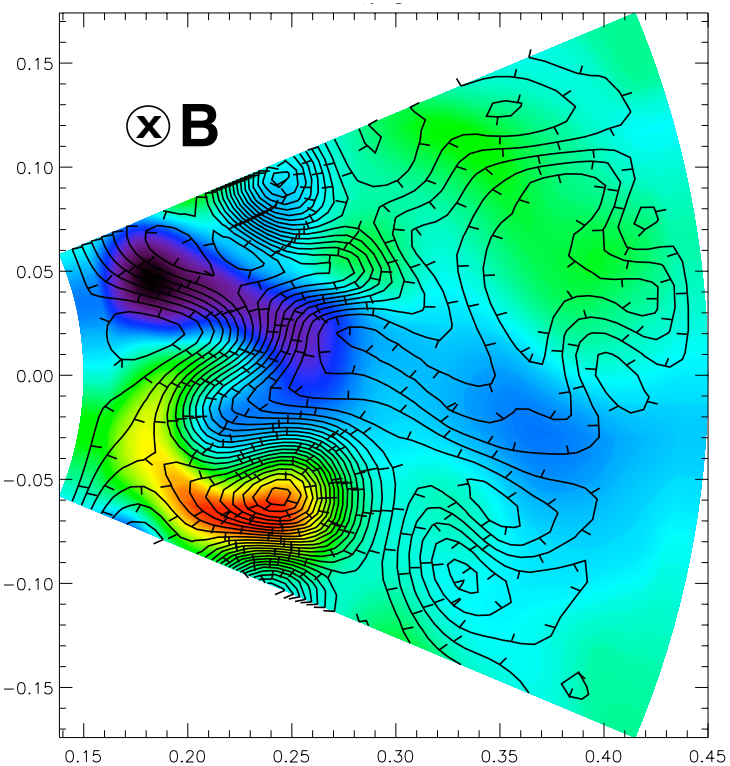
Statistical properties of turbulent fluctuations indicate non-Gaussian behavior

- PDFs of turbulent fluctuations of Γ, N, ϕ all are strongly asymmetric
- Usual statistics – skewness, kurtosis indicate non-Gaussian fluctuations
- In tokamak edge plasma this is often interpreted as non-diffusive transport by density "blobs"
- For LAPD there is no curvature drive that causes blob polarization and transport
- In spite of non-Gaussian features, transport appears consistent with Bohm diffusion
- Do density blobs play a role here?

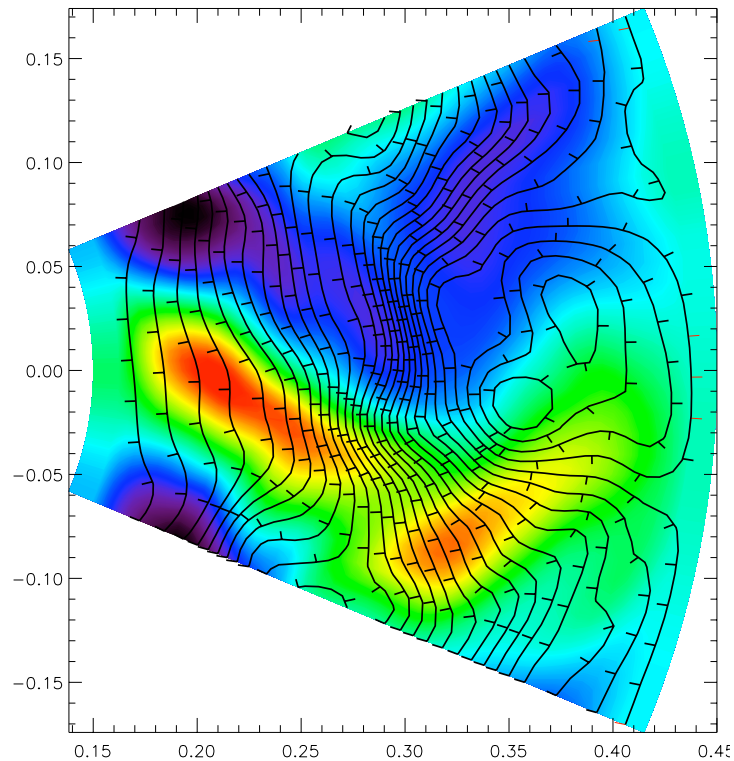


There is evidence of electric polarization of some (but not all) density blobs

This blob appears polarized



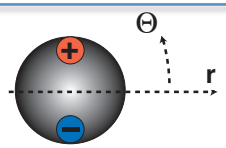
This blob appears non-polarized



Fill-plot for N, line-plot for phi

Polarization of density blobs by centrifugal force is not significant; can be increased by adding radial force

$$\partial_t \varpi = -V_E \bullet \nabla \varpi - \nabla_{\parallel}(N V_{\parallel e}) + b \times \nabla N \bullet \nabla (V_E^2/2) + b \times (F_r / mNT) \bullet \nabla N - \nu_{in} \varpi + \mu \nabla_{\perp}^2 \varpi$$

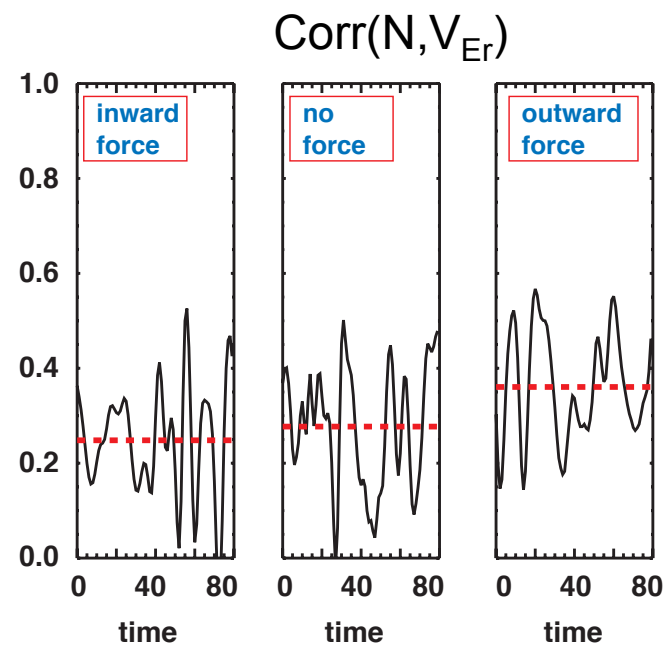


Polarized density blob

- **Ni and V_{Er} perturbations are statistically correlated (otherwise particle flux Γ=0)**

- **Adding radial outward force increases the correlation**
- **Adding radial inward force slightly decreases the correlation**

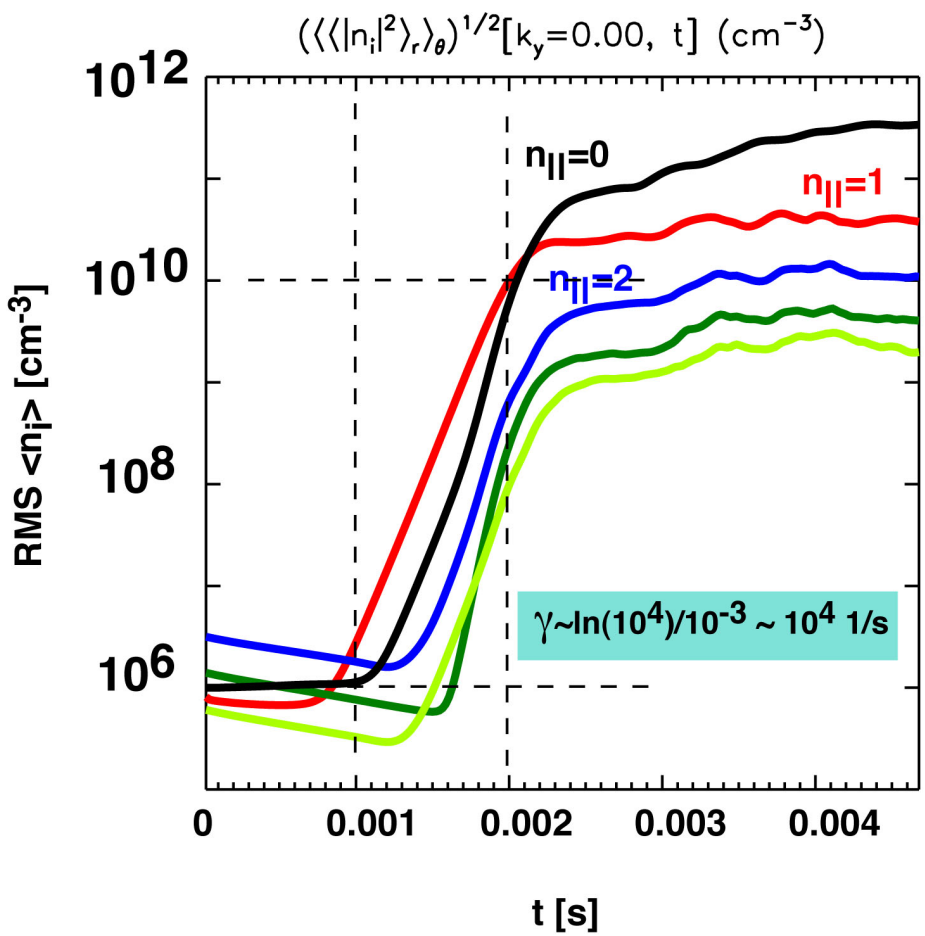
- **Removing V^2 term has little effect => rotation in this case too small for "ballistic" blob transport**



$$corr(X,Y) = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_X}$$

I. Overview of LAPD experiment and BOUT code
II. Physics model and assumptions
III. Linear analysis
IV. General appearance of computed turbulence
V. Comparison with experimental data
VI. Analysis of computed turbulence
VII. Ramifications
VIII. Conclusions

Initially, linear drive comes from finite drift modes $n_{||}$=1 but most fluctuations end up in flute-like mode $n_{||}$=0



- Mode with $n_{||}$=1 comes first
- Growth rate for all modes $\gamma \sim \omega_*$
- Initially $n_{||}$=0 mode tracks $n_{||}$=1
- Later $\gamma_0 \sim 2\gamma_1$ - driven mode?

$$\begin{cases} \dot{\phi}_1 = \gamma_1 \phi_1 \\ \dot{\phi}_0 = C\phi_1^2 \end{cases}$$

- Eventually amplitudes are ordered by $n_{||}$

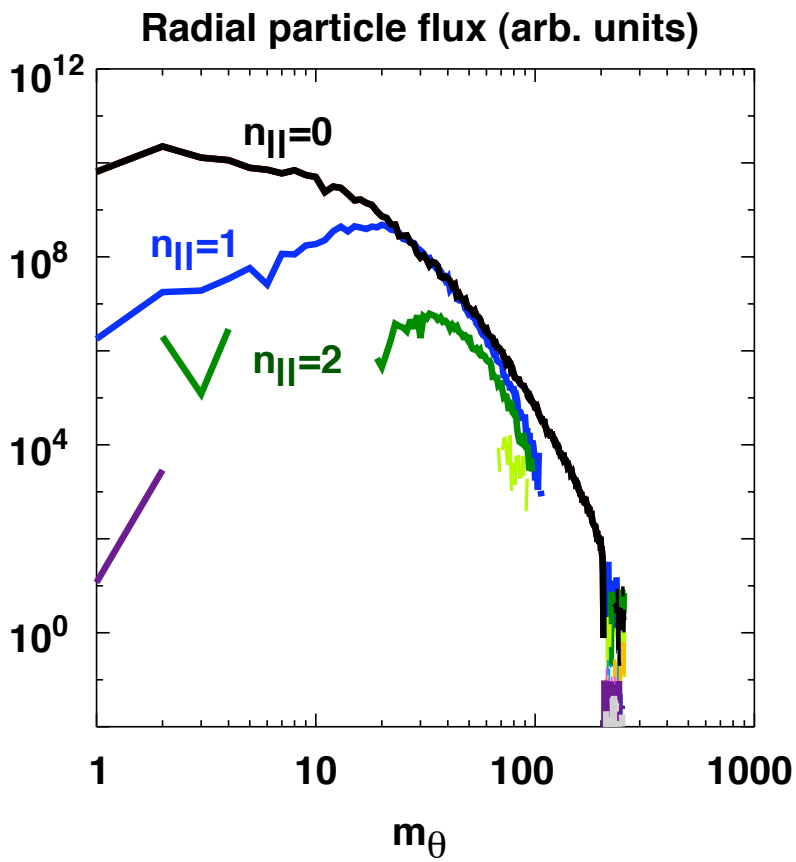
In saturated turbulence, both fluctuations and zonal flow are mostly in flute-like mode $n_{||}=0$

$n_{||}=0$ $n_{||}=1$

$n_i(\rho,\theta)$

$\phi(\rho,\theta)$

Radial particle transport, as well, is mostly caused by flute-like mode $n_{||}=0$



- **Turbulence flux by ExB advection**

$$\Gamma = nV_r = -\frac{c}{rB} n \frac{\partial \phi}{\partial \theta}$$

- **Can be represented by sum of spectral components**

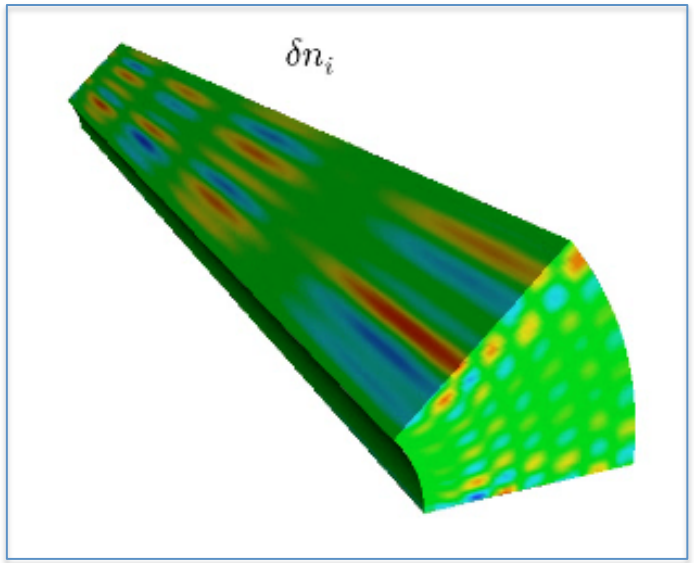
$$\langle \Gamma \rangle_{\theta,||} \propto \sum_k i k_\theta n_k \phi_k^* + cc$$

$$k = (k_\theta, k_||)$$

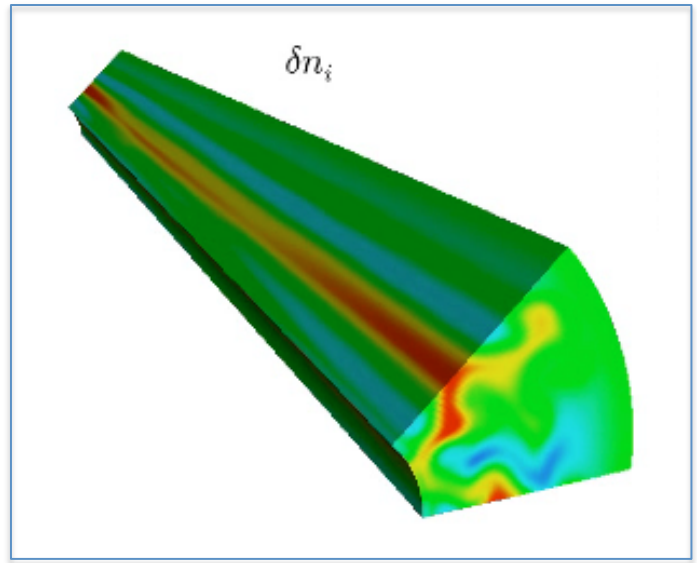
- **Gaps on the plot indicate negative values**

Flute-like density filaments are spontaneously generated in turbulence

Linear phase



Non-linear phase



- **Without perpendicular polarization mechanism (e.g. curvature, centrifugal force) => diffusive transport on the order of Bohm**

- **With perpendicular polarization mechanism => radial advection can become important**

- **Three-wave coupling here is a mechanism of blob generation!**

Summary and conclusions

- Turbulence calculations with 3D reduced collisional fluid plasma model demonstrate qualitative and semi-quantitative similarity to LAPD data - temporal spectra, spatial correlation length, fluctuation amplitude, level of particle flux
- Calculated turbulence has non-Gaussian features; however particle flux appears to be close to Bohm diffusion - consistent with experimental data
- Turbulence k-spectra exhibit direct and inverse cascades in both azimuthal and axial wave-numbers
- Flute-like mode dominates fluctuations, zonal flow, transport
- Coupling between drift-wave and KH modes is a mechanism of density blobs generation
- Work in progress on extending the model (adding T_e, V_{lli}, sheath, E&M terms) to improve further the agreement with LAPD data