Surfing the Waves: a Particle Pusher's Personal Perspective

W. W. Lee
Princeton Plasma Physics Laboratory

Dawson Prize Presentation
22nd International Conference
on Numerical Simulation of Plasmas
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THE PHYSICS OF FLUIDS VOLUME 5, NUMBER 4 APRIL, 1962

One-Dimensional Plasma Model

JOHN DAWSON
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey
(Received June 27, 1961; revised manuscript received December 27, 1961)

A one-dimensional plasma model consisting of a large number of identical charge sheets embedded in a uniform fixed neutralizing background is investigated by following the sheet motions on a high-speed computer. The thermalizing properties and ergodic behavior of the system are examined and found to be in agreement with the assumption that one is equally likely to find the system in equal volumes of the available phase space. The velocity distribution, Debye shielding, drag on fast and slow sheets, diffusion in velocity space, the Landau damping of the Fourier modes, the amplitude distribution function for the Fourier modes, and the distribution of electric fields felt by the sheets were obtained for the plasma in thermal equilibrium and compared with theoretically predicted values. In every case, except one, the drag on a slow sheet, the numerical results agreed with theory to within the statistical accuracy of the results. The numerical results for the drag on a slow sheet were about a factor of 2 lower than the theory predicated indicating that the approximations made in the theory are not entirely valid. An understanding of the cause of the discrepancy might lead to a better understanding of collisional processes in plasmas

Other pioneers are
Oscar Buneman and Ned Birdsall

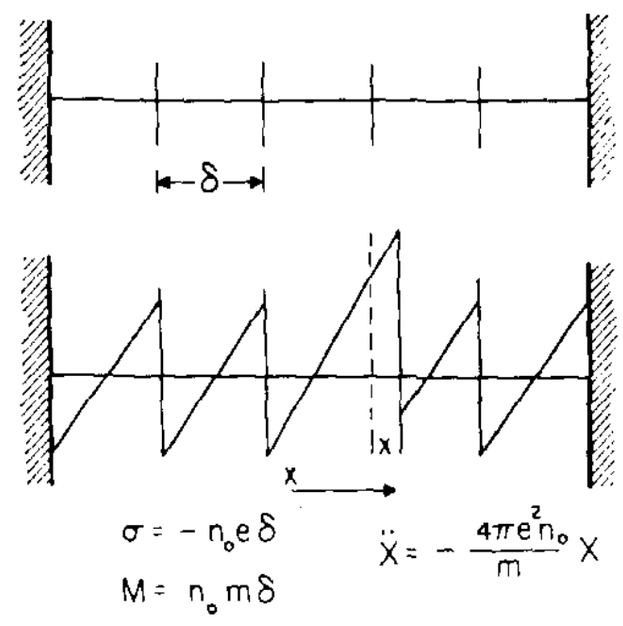

FIG. 1. One-dimensional plasma model.

JOURNAL OF COMPUTATIONAL PHYSICS 6, 247–267 (1970)

Effects of the Spatial Grid in Simulation Plasmas

A. BRUCE LANGDON

Department of Electrical Engineering and Computer Sciences, Electronics Research Laboratory, University of California, Berkeley, California 94720

Received January 22, 1970; revised April 8, 1970

In earlier studies of simulation plasmas the difference schemes for each step of the calculation have been analyzed, but their over-all performance taken together with plasma behavior has not been treated carefully. We begin with a rigorous treatment of the spatial grid, giving here a formulation which includes most codes now in use. This is done in such a way that the role of each step in the calculation is easily identified in the results, and also the expressions for plasma properties are easily compared with the corresponding "real" plasma properties. Details are given for the electrostatic case. The formulation is applied to the question of energy conservation, and to linear wave dispersion and instability. The effect of the spatial grid is to smooth the interaction force somewhat and to couple plasma perturbations to perturbations at other wavelengths, called aliases. The strength of the coupling depends on the smoothness of the interpolation methods used. Its importance depends roughly on how well the plasma would respond, in the absence of the grid, to wavenumbers $k \sim 2\pi/\Delta x$; e.g., if the Debye length is too small the coupling can destabilize plasma oscillations even in a thermal plasma.

- Grid spacing limitation: $\Delta x \leq \lambda_{De}$ -- it's violation will cause numerical instability

- Suppression of plasma waves also alleviates this condition.

JOURNAL OF COMPUTATIONAL PHYSICS 30, 202–221 (1979)

Analysis of the Time Integration in Plasma Simulation

A. BRUCE LANGDON

Lawrence Livermore Laboratory, University of California, Livermore, California 94550

Received December 19, 1977; revised April 6, 1978

This paper treats the *collective* behavior of hot plasma as modified by the numerical time integration methods used to integrate the particle equations of motion in computer simulation of plasmas. No approximation, other than ignoring roundoff errors, is made in analyzing the finite-difference algorithms. Our results reduce simply and exactly to the corresponding results of plasma theory in the limit $\Delta t \to 0$. The possibility of nonphysical instability is considered. The results of this and of previous papers are combined to describe both the spatial and temporal difference algorithms. The theory is generalized to a class of integration schemes, some algorithms are analyzed, and a new example is synthesized. The difficulty of developing algorithms stable at very large time steps is examined. The present analysis may be combined with an earlier rigorous analysis of the spatial grid used for field equations, to develop a kinetic theory of simulation plasmas paralleling that for real plasmas. This theory may be of use in the design and interpretation of computer simulation experiments.

$$\epsilon \equiv 1 + (w_{pe}\Delta t)^2 \sum_{q=0}^{\infty} q \exp\left[i(\omega\Delta t)q - (k\lambda_{De})^2(\omega_{pe}\Delta t)^2 q^2/2\right] = 0$$

$$X_\alpha \equiv \xi_\alpha Z(\xi_\alpha) = -1 + (kv_{t\alpha})^2 \int_o^\infty t \exp\left[i\omega t - \frac{(kv_{t\alpha}t)^2}{2}\right] dt$$ [Dawson '61]

$\omega_{pe}\Delta t \leq 1$ -- it's violation will cause numerical instability

$kv_{te}\Delta t \equiv (k\lambda_{De})(\omega_{pe}\Delta t) \leq 1$ -- it's violation will cause numerical inaccuracy only

Fluctuation-Dissipation Theorem for Weakly Damped Normal Modes

[Klimontovich '67]

- Particle Noise in a Simulation Plasma

$$|e\Phi(k, w_{pe})/T_e|_{th} = \frac{1}{\sqrt{N} k \lambda_{De}} \qquad \text{plasma waves}$$

$$|e\Phi(k, w_s)/T_e|_{th} = \frac{1}{\sqrt{N}} \qquad \text{ion acoustic waves}$$

- N is the number of simulation particles in the wave, not in the Debye shielding volume.

Fluctuations in Simulations with Finite Size Particles

[Many papers by Birdsall, Langdon and Okuda in '70's]

- Linear Dispersion Relation $\epsilon \equiv 1 + |S_k|^2[1 + \xi_e Z(\xi_e) + \tau + \tau\xi_i Z(\xi_i)]/(k\lambda_{De})^2 = 0$

- Finite size particles only affects short wavelength modes by reducing their fluctuation (noise) level .

- For long wavelength modes with $k^2\lambda_D^2 \ll 1$ and $k^2 a^2 \ll 1$, physics is intact.

J. DENAVIT and J. M. WALSH
Department of Mechanical and Nuclear Engineering, Northwestern University, Evanston, Illinois 60201

Comments Plasma Phys. Controlled Fusion
1981, Vol. 6, No. 6 pp. 209–223
0374-2806/81/0606-0209/$06.50/0

Nonrandom Initializations of Particle Codes

While the most common initialization method for particle codes is the use of random numbers, some concepts of number theory suggest the use of nonrandom, low-discrepancy point sets to achieve quieter initializations. What are low-discrepancy point sets, and what are the results of their applications to simple electrostatic particle simulations?

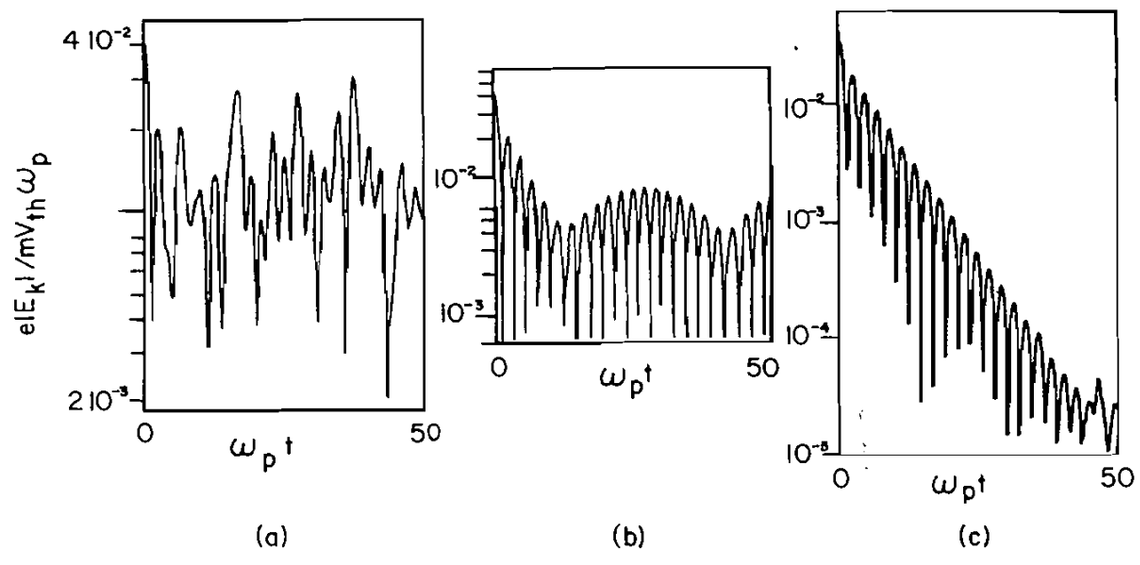

FIGURE 4 Fundamental electric field amplitude $|E_k|$ as a function of time for Landau damping example. (a) Random initialization with 17711 particles. (b) Nonrandom initialization with 17711 particles. (c) Result of a Vlasov simulation.

Reviews of Modern Physics, Vol. 55, No. 2, April 1983

Particle simulation of plasmas

John M. Dawson

Department of Physics, University of California, Los Angeles, California 90024

For plasma with a large number of degrees of freedom, particle simulation using high-speed computers can offer insights and information that supplement those gained by traditional experimental and theoretical approaches. The technique follows the motion of a large assembly of charged particles in their self-consistent electric and magnetic fields. With proper diagnostics, these numerical experiments reveal such details as distribution functions, linear and nonlinear behavior, stochastic and transport phenomena, and approach to steady state. Such information can both guide and verify theoretical modeling of the physical processes underlying complex phenomena. It can also be used in the interpretation of experiments.

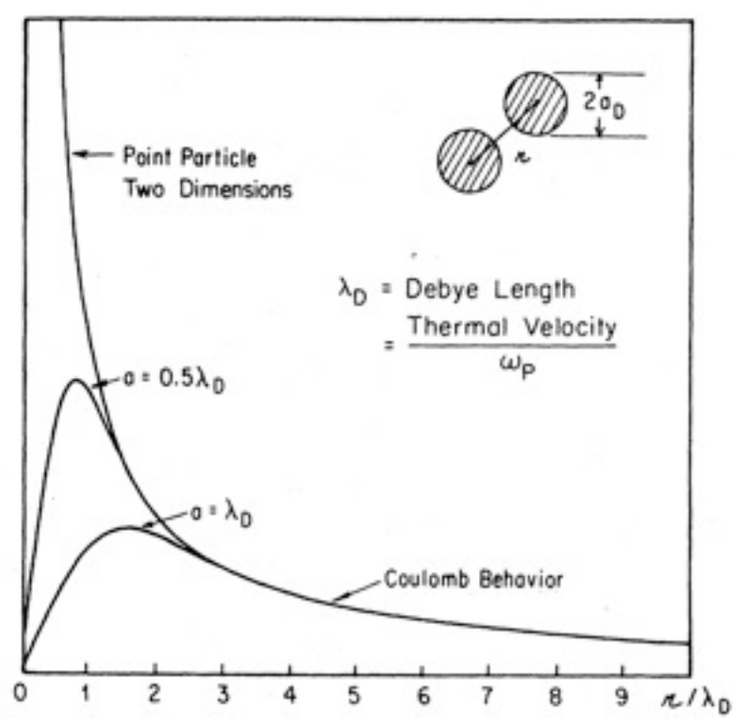

FIG. 2. Force law between finite-size particles in two dimensions for various sized particles. A Gaussian-shaped charge-density profile was used.

JOURNAL OF COMPUTATIONAL PHYSICS **26**, 139–152 (1978)

A Simulation Model for Studying Low-Frequency Microinstabilities

W. W. LEE AND H. OKUDA

Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540

Received March 11, 1976; revised July 30, 1976

A $2\frac{1}{2}$-dimensional, electrostatic particle code in a slab geometry has been developed to study low-frequency oscillations such as drift wave and trapped particle instabilities in a nonuniform bounded plasma. A drift approximation for the electron transverse motion is made which eliminates the high-frequency oscillations at the electron gyrofrequency and its multiples. It is, therefore, possible to study the nonlinear effects such as the anomalous transport of plasmas within a reasonable computing time using a real mass ratio. Several examples are given to check the validity and usefulness of the model, including those using full electron dynamics.

- This is the first attempt to simplify particle dynamics based on gyrating ions and drift kinetic electrons.
- Although more realistic mass ratio can be used, such a simulation model is still not satisfactory,

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Linearized gyro-kinetics

(*Received* 5 *December* 1977)

Abstract—Finite gyroradius effects are retained in a far simpler manner than previous treatments by transforming to the guiding center variables and gyro-averaging *before* introducing magnetic coordinates.

P. J. CATTO

Department of Mechanical and Aerospace Sciences,
University of Rochester,
Rochester, New York 14627,
and
Oak Ridge National Laboratory,
Oak Ridge, Tennessee 37830, *U.S.A.*

$$F(\mathbf{x}, \mathbf{v}, t) \rightarrow F(\mathbf{R}, \mu, U, \varphi, t)$$

$$\mathbf{x} = \mathbf{R} + \rho$$

-- I was alerted to this paper by the late Parvez Guzdar

Phys. Fluids, Vol. 26, No. 2, February 1983

Gyrokinetic approach in particle simulation

W. W. Lee
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08544

(Received 16 October 1981; accepted 20 October 1982)

A new scheme for particle simulation based on the gyrophase-averaged Vlasov equation has been developed. It is suitable for studying linear and nonlinear low-frequency microinstabilities and the associated anomalous transport in magnetically confined plasmas. The scheme retains the gyroradius effects but not the gyromotion; it is, therefore, far more efficient than conventional ones. Furthermore, the reduced Vlasov equation is also amenable to analytical studies.

$$\frac{\partial\langle F\rangle}{\partial t}+\mathbf{v}_{\parallel}\cdot\frac{\partial\langle F\rangle}{\partial\mathbf{R}}-\frac{q}{m}\frac{1}{\Omega}\frac{\partial\langle\Psi\rangle}{\partial\mathbf{R}}\times\hat{\mathbf{b}}\cdot\frac{\partial\langle F\rangle}{\partial\mathbf{R}}-\frac{q}{m}\frac{\partial\langle\Psi\rangle}{\partial\mathbf{R}}\cdot\hat{\mathbf{b}}\frac{\partial\langle F\rangle}{\partial v_{\parallel}}=0, \tag{21}$$

Nonlinear gyrokinetic Vlasov equation

$$\langle\Psi\rangle\equiv\langle\Phi\rangle+\frac{1}{2}\frac{q}{T}\langle\Phi\rangle^2-\frac{1}{2}\frac{q}{T}\langle\Phi^2\rangle$$
$$\simeq\langle\Phi\rangle-\frac{1}{4}\frac{q}{T}\left(\frac{v_\perp}{\Omega}\right)^2\left|\frac{\partial\Phi}{\partial\mathbf{R}_\perp}\right|^2.$$

$$\nabla^2\Phi-k_{Di}^2(n_i/n_0)(\Phi-\tilde{\Phi})=-4\pi e(n_i-n_e), \tag{37}$$

Gyrokinetic Poisson's equation

$$-k_{Di}^2(\Phi-\tilde{\Phi})\sim(\omega_{pi}^2/\Omega_i^2)\nabla_\perp^2\Phi,$$

Density response due to polarization drift

JOURNAL OF COMPUTATIONAL PHYSICS 72, 243–269 (1987)

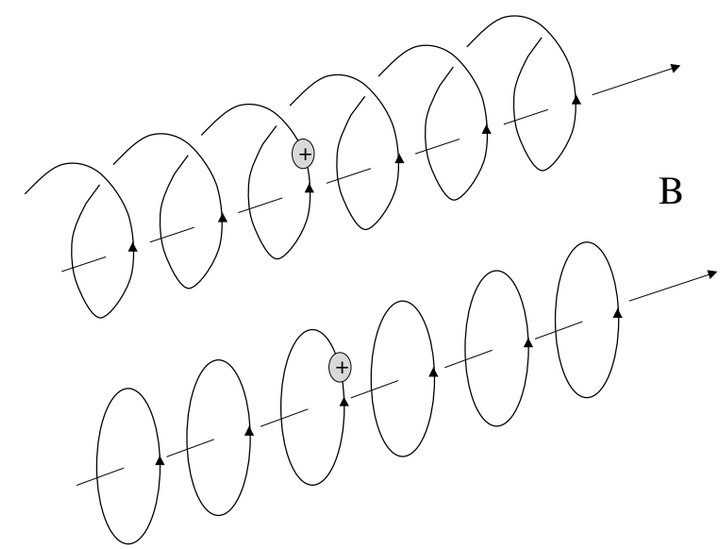
Gyrokinetic Particle Simulation Model

W. W. LEE

Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08544

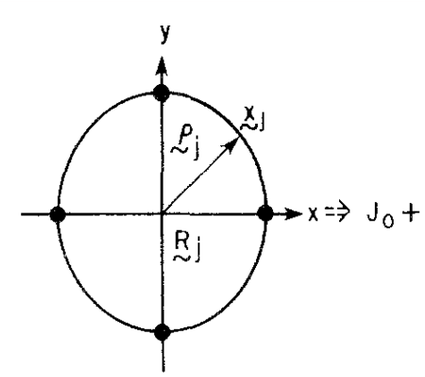
Received June 27, 1986; revised December 19, 1986

A new type of particle simulation model based on the gyrophase-averaged Vlasov and Poisson equations is presented. The reduced system, in which particle gyrations are removed from the equations of motion while the finite Larmor radius effects are still preserved, is most suitable for studying low frequency microinstabilities in magnetized plasmas. The resulting gyrokinetic plasma is intrinsically quasineutral for $\lambda_D \ll \rho_s$ ($\equiv \rho_i (T_e/T_i)^{1/2}$). Thus, without the troublesome space charge waves in the simulation, we can afford to use much larger time steps ($\omega_H \Delta t \lesssim 1$) and grid spacings ($\Delta x_\perp/\rho_s \lesssim 1$) at a much reduced noise level than we would have for conventional particle codes, where $\omega_H \equiv (k_\parallel/k_\perp)(\lambda_D/\rho_s)\,\omega_{pe}$, and $k_\parallel \ll k_\perp$. Furthermore, it is feasible to simulate an elongated system ($L_\parallel \gg L_\perp$) with a three-dimensional grid using the present model without resorting to the usual mode expansion technique, since there is essentially no restriction on the size of $\Delta x_\parallel$ in a gyrokinetic plasma. The new approach also enables us to further separate the time and spatial scales of the simulation from those associated with global transport through the use of multiple spatial scale expansion. Thus, the model can be a very efficient tool for studying anomalous transport problems related to steady-state drift-wave turbulence in magnetic confinement devices. It can also be applied to other areas of plasma physics. © 1987 Academic Press, Inc



$$\langle \exp(i\mathbf{k}\cdot\boldsymbol{\rho})\rangle_\varphi = \sum_{n=-\infty}^{\infty} J_n(k_\perp \rho) \sum_{l=1}^{M} \exp(i2\pi nl/M)/M$$

$$= \sum_{n=-\infty}^{\infty} J_n(k_\perp \rho) \sin(2\pi n) \cot(n\pi/M)/2M. \qquad (19)$$



M=1	M=2	M=3	M=4
J_1	0	0	0
J_2	J_2	0	0
J_3	J_3	J_3	0
J_4	J_4	J_4	J_4

FIG 1. Numerical scheme for coordinate transformation for the jth particle.

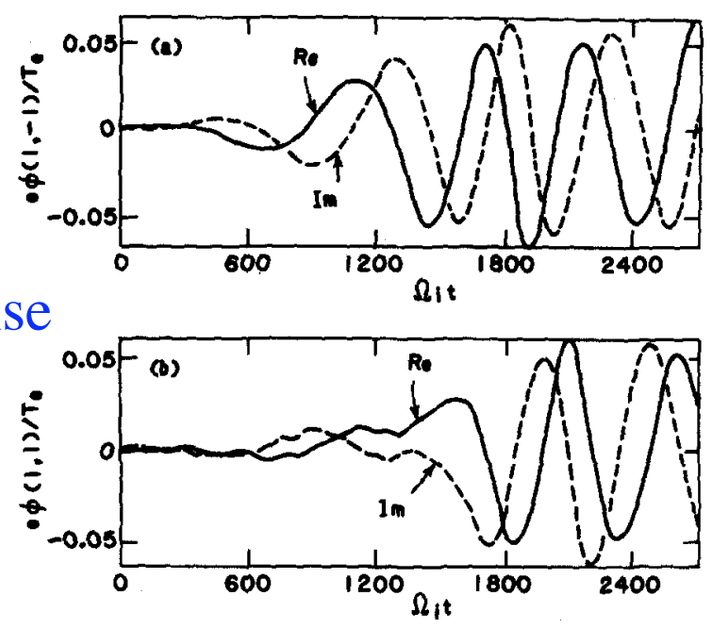
Gyrokinetic particle simulation of ion temperature gradient drift instabilities

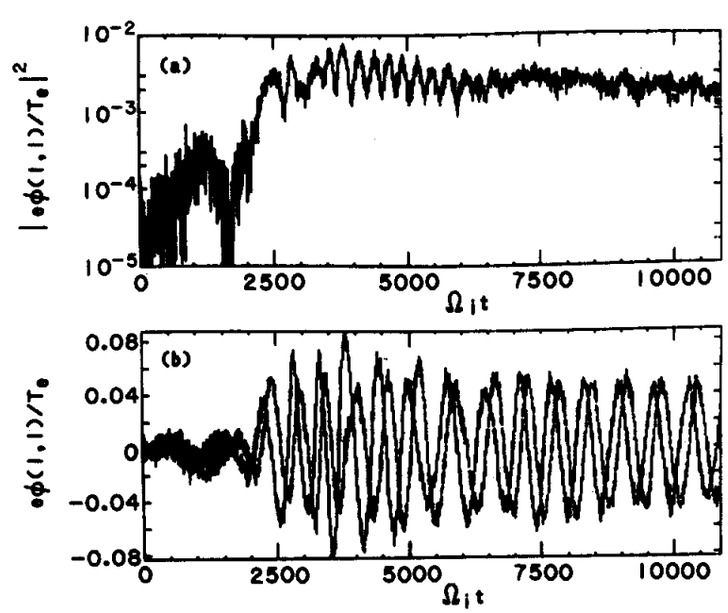
W. W. Lee and W. M. Tang
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08544

(Received 30 March 1987; accepted 3 December 1987)

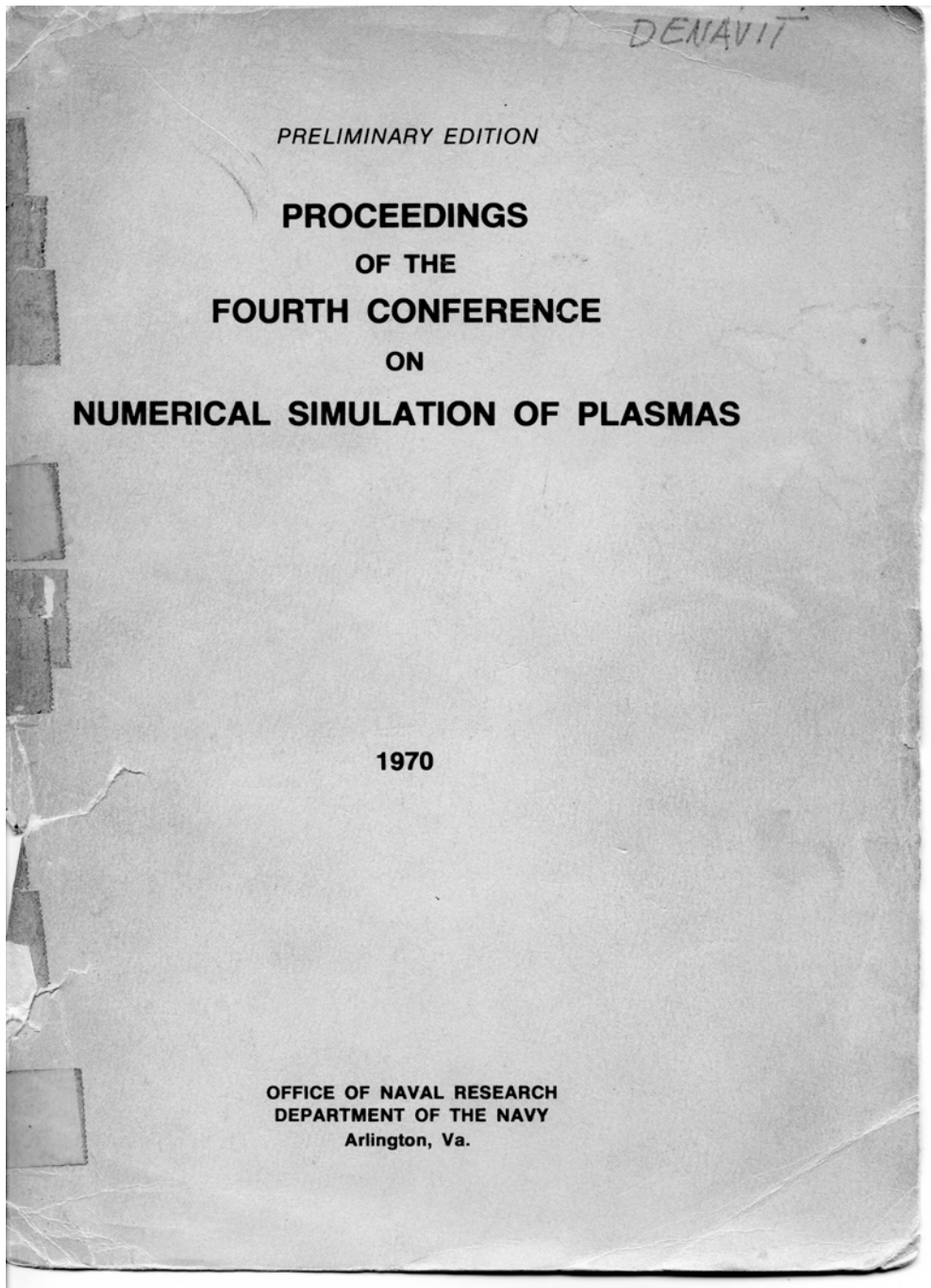
Ion temperature gradient drift instabilities have been investigated using gyrokinetic particle simulation techniques for the purpose of identifying the mechanisms responsible for their nonlinear saturation as well as the associated anomalous transport. For simplicity, the simulation has been carried out in a shear-free slab geometry, where the background pressure gradient is held fixed in time to represent quasistatic profiles typical of tokamak discharges. It is found that the nonlinearly generated zero-frequency responses for the ion parallel momentum and pressure are the dominant mechanisms giving rise to saturation. This is supported by the excellent agreement between the simulation results and those obtained from mode-coupling calculations, which give the saturation amplitude as $|e\Phi/T_e| \cong (|\omega_l + i\gamma_l|/\Omega_i)/(k_\perp \rho_s)^2$, and the quasilinear thermal diffusivity as $\chi_i \cong \gamma_l/k_\perp^2$, where ω_l and γ_l are the linear frequency and growth rate, respectively, for the most unstable mode of the system. In the simulation, the time evolution of χ_i after saturation is characterized by its slow relaxation to a much lower level of thermal conduction. On the other hand, a small amount of electron–ion collisions, which has a negligible effect on the linear stability, can cause significant enhancement of χ_i in the steady state.

Adiabatic Electron Response





Drift Kinetic Electrons - noisy



Noise Suppression Techniques in Macroparticle Models of Collisionless Plasmas*

J. A. Byers
Lawrence Radiation Laboratory
University of California
Livermore, California

ABSTRACT

Noise reduction techniques are examined as applied to macro particle computer models of collisionless plasmas. The quiet start technique, particle weighting, and the usefulness of forcing the system to a linear one are discussed.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

Linearized Trajectory Method

[J. Byers, Fourth Conference on Numerical Simulation of Plasmas, 1970]

- In my opinion, this paper is the forerunner of the modern-day delta-f schemes.

$$\mathbf{x}_{\alpha j} = \mathbf{x}_{\alpha j0} + \mathbf{x}_{\alpha j1}\,, \qquad \mathbf{v}_{\alpha j} = \mathbf{v}_{\alpha j0} + \mathbf{v}_{\alpha j1}$$

$$F_{\alpha 0} = \sum_{j=1}^{N} \delta(\mathbf{x} - \mathbf{x}_{\alpha j0})\delta(\mathbf{v} - \mathbf{v}_{\alpha j0})$$

$$\delta f_\alpha = -\sum_{j=1}^{N} \left(\mathbf{x}_{\alpha j1} \cdot \frac{\partial}{\partial \mathbf{x}} + \mathbf{v}_{\alpha j1} \cdot \frac{\partial}{\partial \mathbf{v}} \right) \delta(\mathbf{x} - \mathbf{x}_{\alpha j0})\delta(\mathbf{v} - \mathbf{v}_{\alpha j0})$$

$$\frac{d\mathbf{x}_{\alpha j0}}{dt} = \mathbf{v}_{\alpha j0}, \qquad \frac{d\mathbf{v}_{\alpha j0}}{dt} = 0,$$

$$\frac{d\mathbf{x}_{\alpha j1}}{dt} = \mathbf{v}_{\alpha j1}, \qquad \frac{d\mathbf{v}_{\alpha j1}}{dt} = \frac{q_\alpha}{m_\alpha}\mathbf{E}(\mathbf{x}_{\alpha j0})$$

$$\delta n_\alpha = -\frac{\partial}{\partial \mathbf{x}} \cdot \sum_{j=1}^{N} \mathbf{x}_{\alpha j1}\delta(\mathbf{x} - \mathbf{x}_{\alpha j0})$$

$$\nabla \cdot \mathbf{E} = 4\pi \sum_\alpha q_\alpha \delta n_\alpha$$

$$\delta n_\alpha = -\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{x}_{\alpha 1} \sum_{j=1}^{N} \delta(\mathbf{x} - \mathbf{x}_{\alpha j0}) = -\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{x}_{\alpha 1} n_{\alpha 0}$$

$$\frac{\partial}{\partial t}\delta n_\alpha = -\frac{\partial}{\partial \mathbf{x}} \cdot \left(\frac{\partial \mathbf{x}_{\alpha 1}}{\partial t} n_{\alpha 0} \right) = -\frac{\partial}{\partial \mathbf{x}} \cdot \sum_{j=1}^{N} \mathbf{v}_{\alpha j1}\delta(\mathbf{x} - \mathbf{x}_{\alpha j0})$$

$$\nabla^2 \frac{\partial \phi}{\partial t} = 4\pi \sum_\alpha q_\alpha \frac{\partial}{\partial \mathbf{x}} \cdot \sum_{j=1}^{N} \mathbf{v}_{\alpha j1}\delta(\mathbf{x} - \mathbf{x}_{\alpha j0})$$

Compare this later to the split-weight scheme

Proceedings of the Fourth Conference on Numerical Simulation of Plasmas, Naval Research Laboratory, Arlington, VA, 1970

Numerical Solutions of the Vlasov Equation

J. Denavit
Plasma Physics Division
Naval Research Laboratory
Washington, D.C.

ABSTRACT

This paper present a review of numerical solutions of the Vlasov equation by the Fourier-Fourier transform method and by the direct method of integration in which the distribution function is pushed in the phase plane. Computer codes were written to implement both methods and numerical solutions of two-stream instability problems by both methods are compared. The Fourier-Fourier transform code uses fast Fourier transforms to compute the convolution term which appears in the transformed Vlasov equation. This technique allows solutions with a large number of modes. In the direct integration code, the distribution function is not generally reconstructed at every time step. The examples presented in the paper suggest that, by properly choosing the frequency of reconstruction of the distribution function, it may be possible to minimize both the noise due to discrete particle interactions and the diffusion which occurs every time the distribution function is reconstructed.

Vlasov simulation has no noise problem, but it has other numerical difficulties in higher dimensions.

Partially Linearized Algorithms in Gyrokinetic Particle Simulation

A. M. DIMITS* AND W. W. LEE

Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543

Received August 30, 1990; revised September 4, 1992

In this paper, particle simulation algorithms with time-varying weights for the gyrokinetic Vlasov–Poisson system have been developed. The primary purpose is to use them for the removal of the selected nonlinearities in the simulation of gradient-driven microturbulence so that the relative importance of the various nonlinear effects can be assessed. It is hoped that the use of these procedures will result in a better understanding of the transport mechanisms and scaling in tokamaks. Another application of these algorithms is for the improvement of the numerical properties of the simulation plasma. For instance, implementations of such algorithms (1) enable us to suppress the intrinsic numerical noise in the simulation, and also (2) make it possible to regulate the weights of the fast-moving particles and, in turn, to eliminate the associated high frequency oscillations. Examples of their application to drift-type instabilities in slab geometry are given. We note that the work reported here represents the first successful use of the weighted algorithms in particle codes for the nonlinear simulation of plasmas. © 1993 Academic Press, Inc.

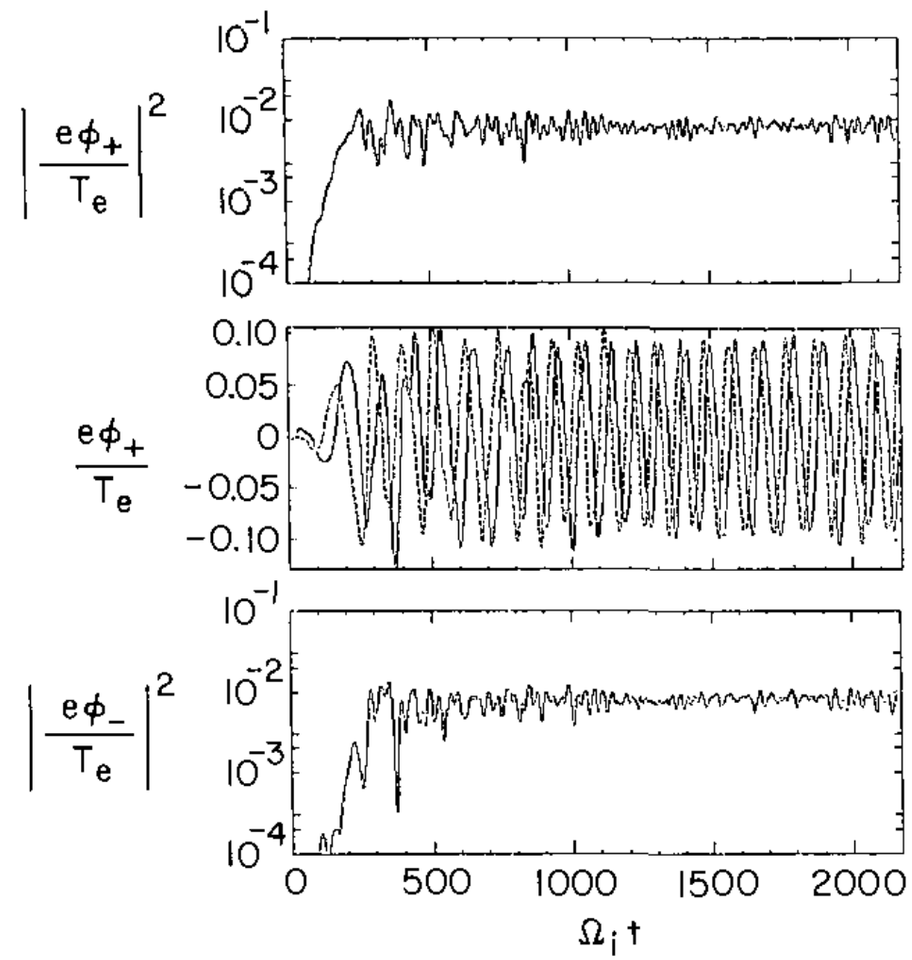


FIG. 2. Time evolution of $\phi_\pm$ using the linearized-$E_\parallel$ algorithm of Section 4.1 for $\theta = 0.002$, $\tau = 100$, $\nu_{ei} = 0.001$, and $\Delta t = 1.09$.

A fully nonlinear characteristic method for gyrokinetic simulation

S. E. Parker and W. W. Lee
Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543

(Received 17 June 1992; accepted 16 September 1992)

A new scheme that evolves the perturbed part of the distribution function along a set of characteristics that solves the fully nonlinear gyrokinetic equations is presented. This low-noise nonlinear characteristic method for particle simulation is an extension of the partially linear weighting scheme, and may be considered an improvement over existing δf methods. Some of the features of this new method include the ability to keep all nonlinearities, particularly those associated with the velocity space, the use of conventional particle loading techniques, and also the retention of the conservation properties of the original gyrokinetic system in the numerically converged limit. The new method is used to study a one-dimensional drift wave model that isolates the parallel velocity nonlinearity. A mode coupling calculation for the saturation amplitude is given, which is in good agreement with the simulation results. Finally, the method is extended to the electromagnetic gyrokinetic equations in general geometry.

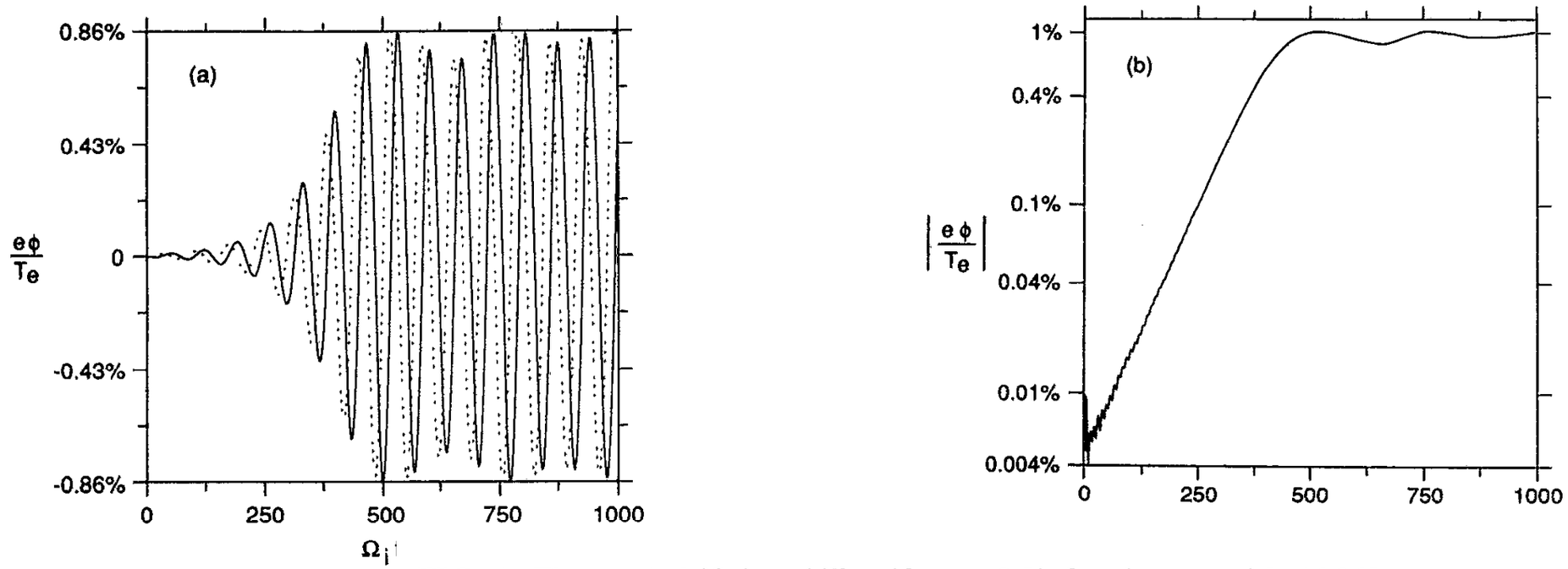


FIG. 4. The $n=1$ drift instability ($k_\perp \rho_i \simeq 0.8$) for the run with 46 368 particles on a 64-grid system. (a) The time history for the real (solid line) and imaginary (dashed line) parts of the electrostatic potential and (b) the corresponding amplitude evolution.

The split-weight particle simulation scheme for plasmas

Igor Manuilskiy and W. W. Lee
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543

(Received 10 November 1999; accepted 14 January 2000)

An efficient numerical method for treating electrons in magnetized plasmas has been developed. The scheme, which is based on the perturbative (δf) gyrokinetic particle simulation, splits the particle electron responses into adiabatic and nonadiabatic parts. The former is incorporated into the gyrokinetic Poisson's equation, while the latter is calculated dynamically with the aid of the charge conservation equation. The new scheme affords us the possibility of suppressing unwanted high-frequency oscillations and, in the meantime, relaxing the Courant condition for the thermal particles moving in the parallel direction. It is most useful for studying low-frequency phenomena in plasmas. As an example, one-dimensional drift wave simulation has been carried out using the scheme and the results are presented in this paper. This methodology can easily be generalized to problems in three-dimensional toroidal geometry, as well as those in unmagnetized plasmas.
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Method for solving the gyrokinetic Poisson equation in general geometry

Z. Lin and W. W. Lee
Princeton Plasma Physics Laboratory, Princeton University, P.O. Box 451, Princeton, New Jersey 08543-0451
(Received 16 February 1995; revised manuscript received 24 May 1995)

A generalized gyrokinetic Poisson solver has been developed and implemented in gyrokinetic particle simulation of low frequency microinstabilities in magnetic fusion plasmas. This technique employs local operations in the configuration space to compute the polarization density response and automatically takes into account the background profile effects contained in the gyrokinetic Poisson equation. It is useful for the global gyrokinetic simulation of magnetized plasmas in general equilibria, where the traditional spectral method is not applicable. The numerical scheme is also most amenable to massively parallel algorithms.

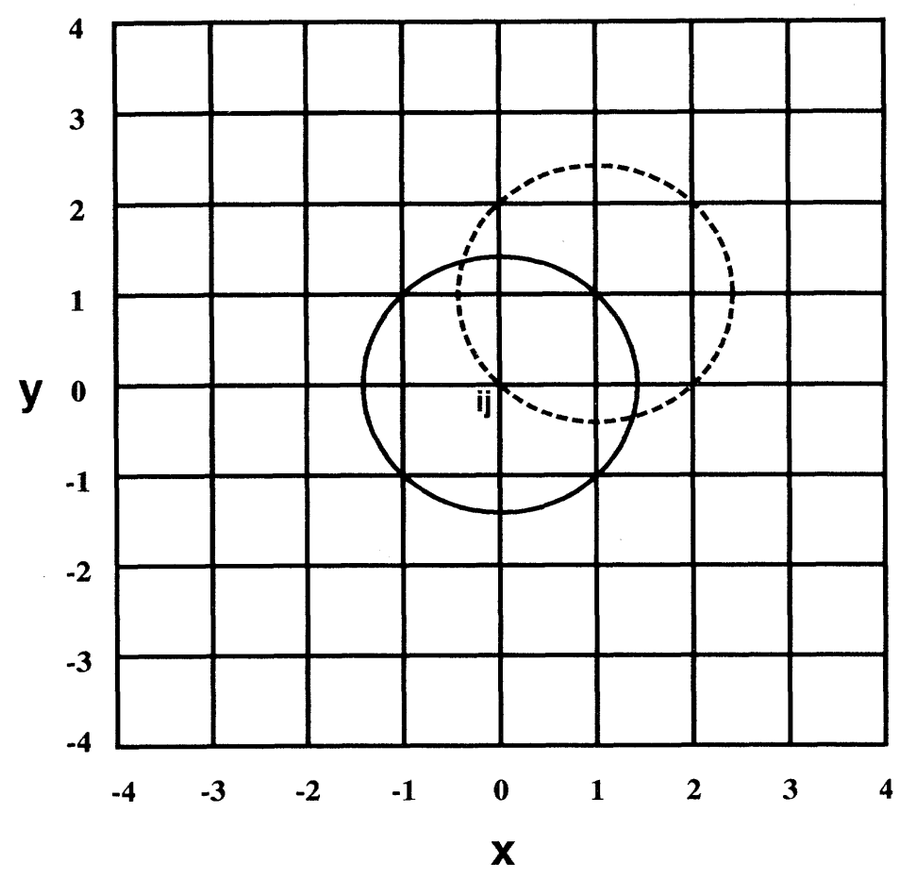

FIG. 1. Schematic procedure for the calculation of $\tilde{\phi}$ using one δ function. The length unit is ρ_i.

Gyrokinetic Simulation of Ion Temperature Gradient Driven Turbulence in 3D Toroidal Geometry

S. E. Parker, W. W. Lee, and R. A. Santoro

Princeton Plasma Physics Laboratory, Princeton University, P.O. Box 451, Princeton, New Jersey 08543

(Received 22 April 1993)

Results from a fully nonlinear three-dimensional toroidal electrostatic gyrokinetic simulation of the ion temperature gradient instability are presented. The model has adiabatic electrons and the complete gyrophase-averaged ion dynamics, including trapped particles. Results include the confirmation of the radially elongated ballooning mode structure predicted by linear theory, and the nonlinear saturation of these toroidal modes. The ensuing turbulent spectrum retains remnants of the linear mode structure, and has very similar features as recent experimental fluctuation measurements.

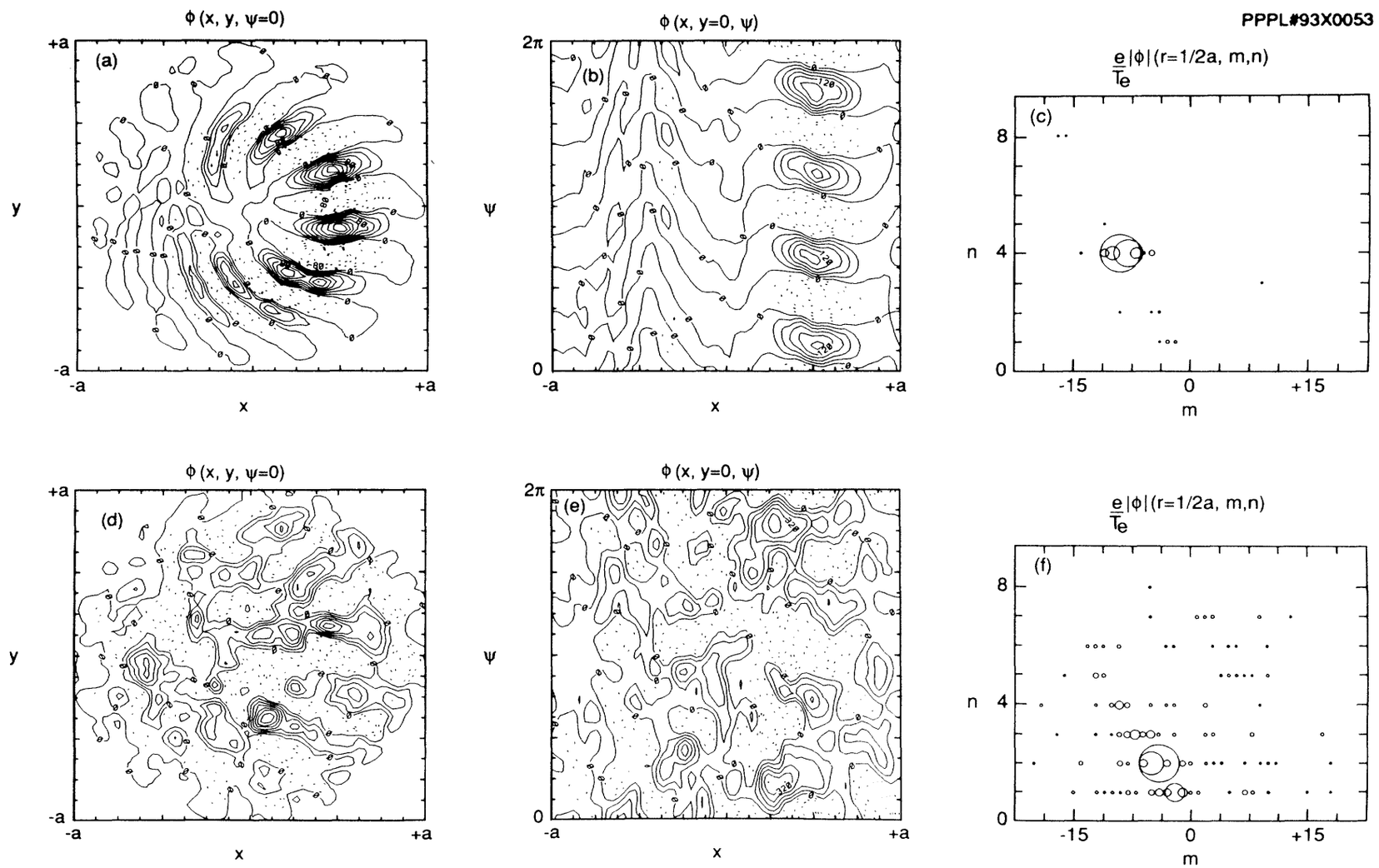

FIG. 1. Plots of the electrostatic potential during the linear phase and nonlinearly saturated steady state. (a) Poloidal cross section during the linear phase, (b) toroidal cross section during the linear phase, (c) excited toroidal and poloidal harmonics during the linear phase; the size of the circle indicates amplitude of the potential; measurement is made at the $q=2$ surface; (d)–(f) are the same diagnostics, taken during the saturated steady state.

Turbulent Transport Reduction by Zonal Flows: Massively Parallel Simulations

Z. Lin,* T. S. Hahm, W. W. Lee, W. M. Tang, R. B. White

Three-dimensional gyrokinetic simulations of microturbulence in magnetically confined toroidal plasmas with massively parallel computers showed that, with linear flow damping, an asymptotic residual flow develops in agreement with analytic calculations. Nonlinear global simulations of instabilities driven by temperature gradients in the ion component of the plasma support the view that turbulence-driven fluctuating zonal flows can substantially reduce turbulent transport. Finally, the outstanding differences in the flow dynamics observed in global and local simulations are found to be due to profile variations.

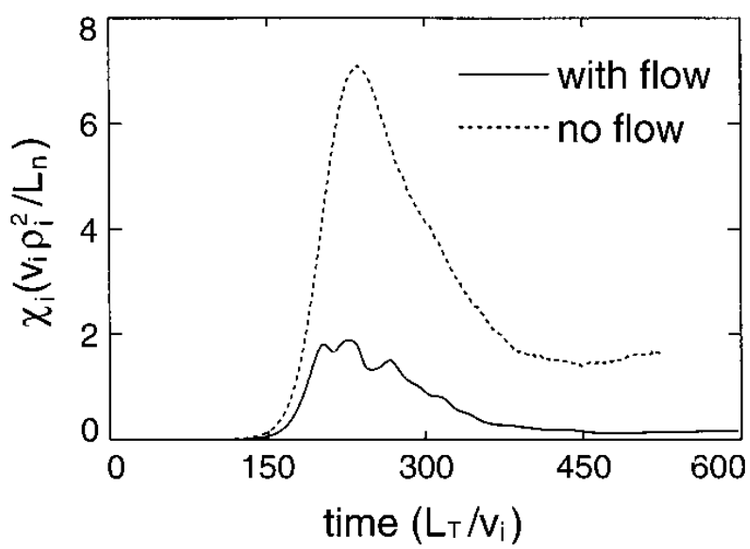

Fig. 1. Time history of ion heat conductivities with (solid) and without (dotted) **E** × **B** flows in global simulations with realistic plasma parameters.

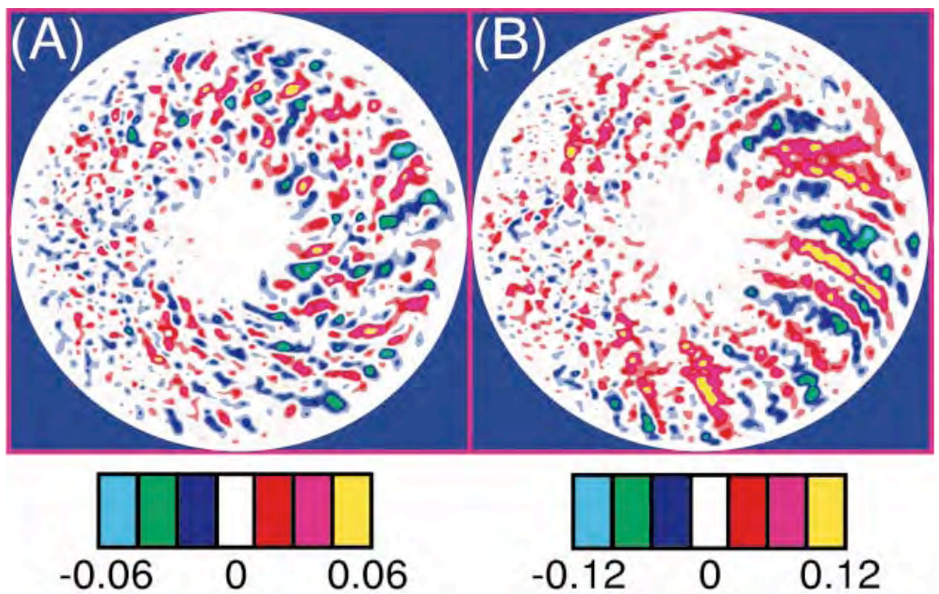

Fig. 2. Poloidal contour plots of fluctuation potential ($e\Phi/T_i$) in the steady state of nonlinear global simulation with **E** × **B** flows included (**A**) and with the flows suppressed (**B**). The dominant poloidal spectrum $k_\theta = 0$ mode is filtered out to highlight the differences in the turbulent eddy size.

COMPUTATIONAL SCIENCE & DISCOVERY

Nonlinear turbulent transport in magnetic fusion plasmas

W W Lee, S Ethier, R Kolesnikov, W X Wang and W M Tang

Plasma Physics Laboratory, Princeton University, Princeton, NJ 08543, USA
E-mail: wwlee@pppl.gov

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Abstract. For more than a decade, the study of microturbulence driven by ion temperature gradient (ITG) drift instabilities in tokamak devices has been an active area of research in magnetic fusion science for both experimentalists and theorists alike. An important impetus for this avenue of research was the discovery of the radial streamers associated with the ITG modes in the early 1990s using the particle-in-cell (PIC) simulation method. Subsequently, ITG simulations using codes with increasing realism have been made possible by the dramatic increase in computing power. Notable examples were the demonstration of the importance of nonlinearly generated zonal flows in regulating ion thermal transport and the transition from Bohm to gyroBohm scaling with increased device size. In this paper, we will describe an interesting nonlinear physical process, as well as the resulting turbulent transport, that is associated with the interactions between the nonlinear parallel acceleration of the ions and the zonal flow modes. This study was carried out by utilizing a fully parallelized three-dimensional PIC code in global toroidal geometry on the most advanced, modern, massively parallel supercomputers.

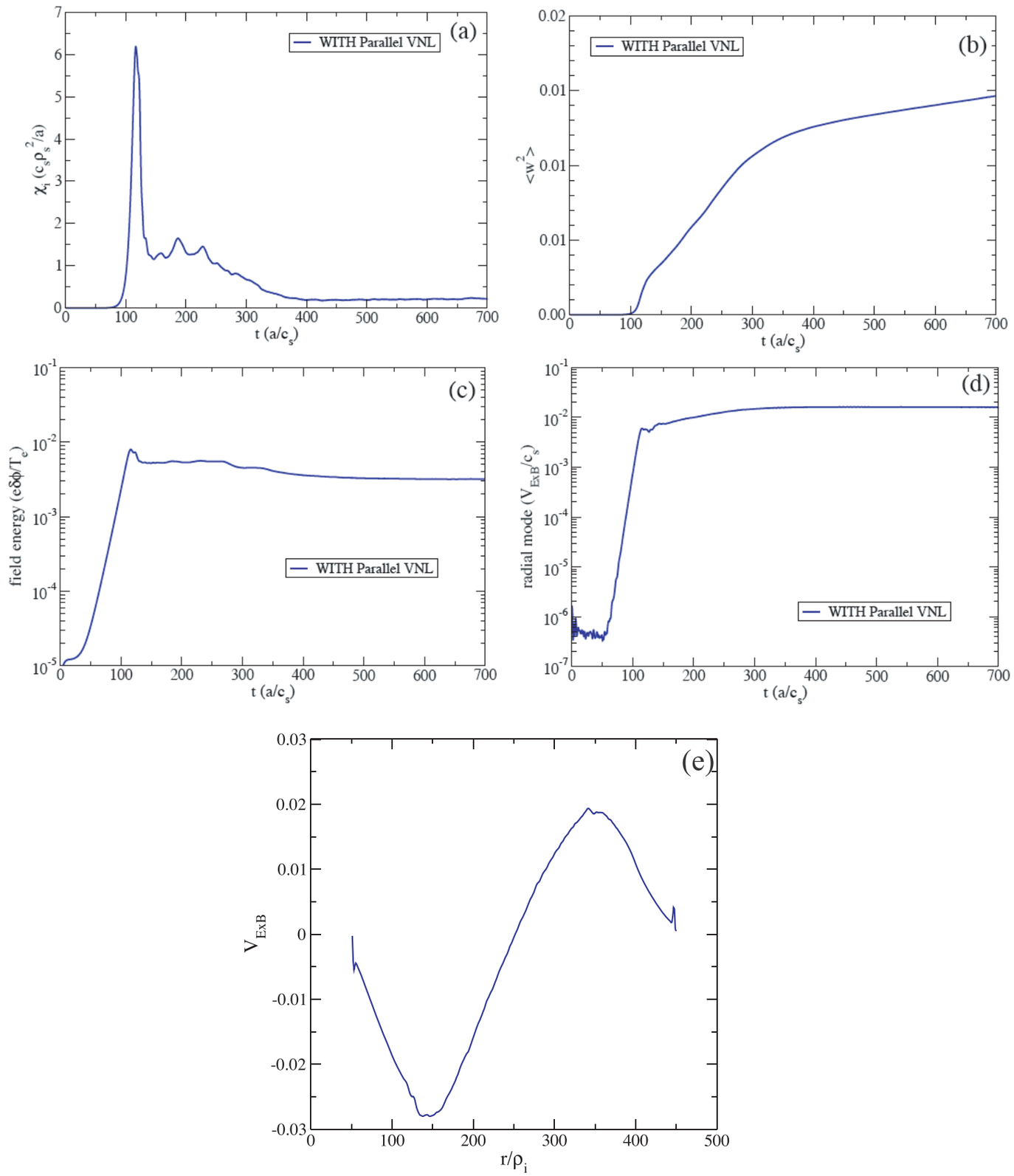

Figure 5. Time evolution of (a) the ion thermal flux, (b) the particle weights, (c) the field energy, and (d) the radial modes, as well as (e) the zonal flow structure for $a/\rho = 500$ including both the nonlinearly generated zonal flows and the velocity space nonlinearity (VNL).

Shear-Alfvén waves in gyrokinetic plasmas

W. W. Lee, J. L. V. Lewandowski, T. S. Hahm, and Z. Lin
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543

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It is found that the thermal fluctuation level of the shear-Alfvén waves in a gyrokinetic plasma is dependent on plasma $\beta(\equiv c_s^2/v_A^2)$, where c_s is the ion acoustic speed and v_A is the Alfvén velocity. This unique thermodynamic property based on the fluctuation–dissipation theorem is verified in this paper using a new gyrokinetic particle simulation scheme, which splits the particle distribution function into the equilibrium part as well as the adiabatic and nonadiabatic parts. The numerical implication of this property is discussed. © *2001 American Institute of Physics*.

[DOI: 10.1063/1.1400124]

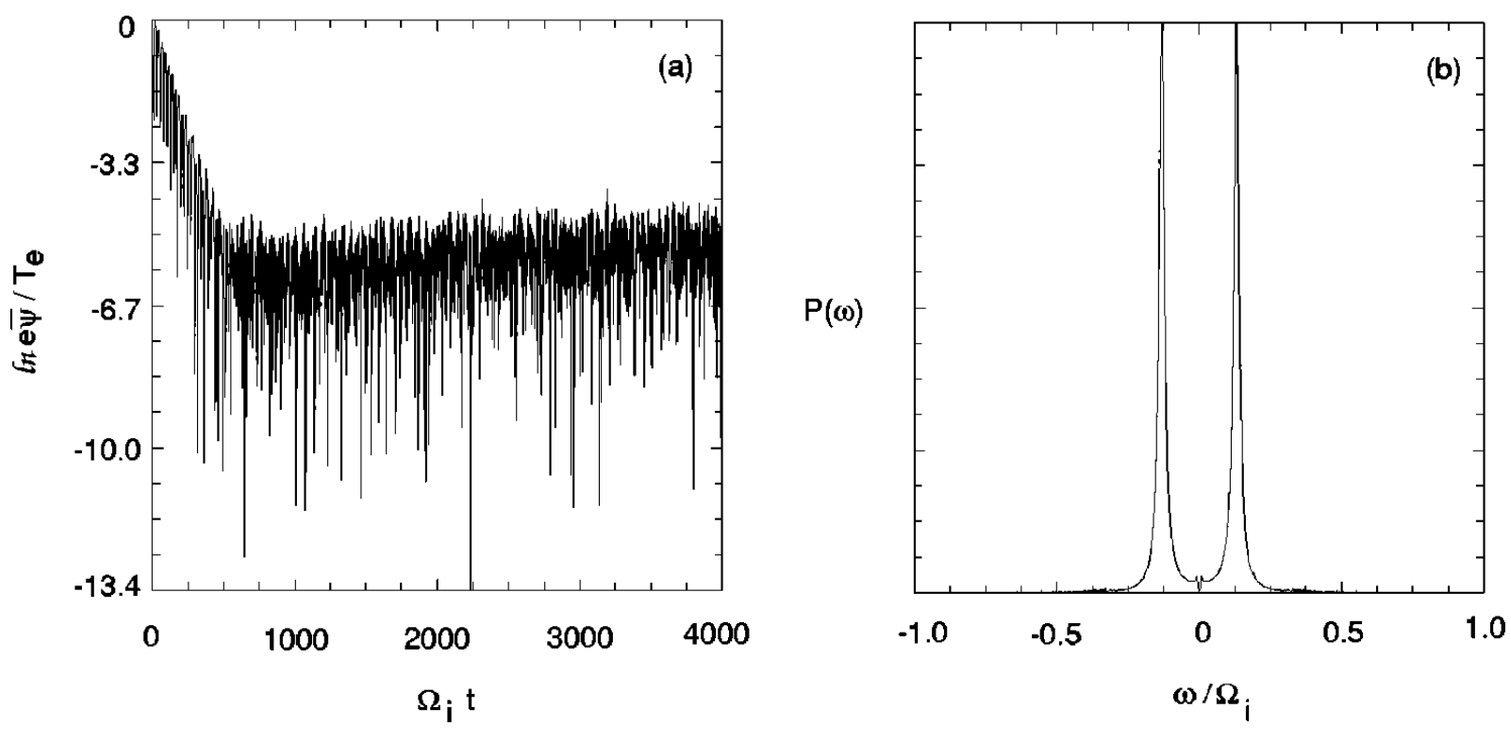

FIG. 2. (a) Damping rate and noise level, and (b) frequencies for the shear-Alfvén waves for $\beta=0.1\%$.

Alfvén waves in gyrokinetic plasmas

W. W. Lee and H. Qin
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543

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A brief comparison of the properties of Alfvén waves that are based on the gyrokinetic description with those derived from the magnetohydrodynamics (MHD) equations is presented. The critical differences between these two approaches are the treatment of the ion polarization effects. As such, the compressional Alfvén waves in a gyrokinetic plasma can be eliminated through frequency ordering, whereas geometric simplifications are needed to decouple the shear Alfvén waves from the compressional Alfvén waves within the context of MHD. Theoretical and numerical procedures of using gyrokinetic particle simulation for studying microturbulence and kinetic-MHD physics including finite Larmor radius effects are also presented. © *2003 American Institute of Physics*. [DOI: 10.1063/1.1590666]

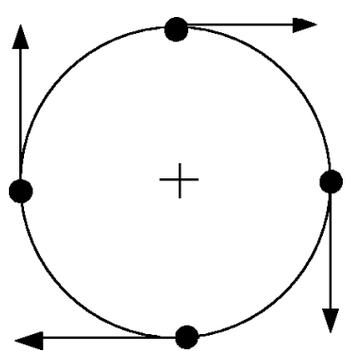

FIG. 1. Four-point approximation for a rotating ion ring.

$$\langle \mathbf{v}_\perp e^{-i\mathbf{k}\cdot\boldsymbol{\rho}_{\alpha j}} \rangle_\varphi = \sum_{n=-\infty}^{\infty} J_n(k_\perp \rho_{\alpha j}) e^{-in\theta} \frac{v_\perp}{L}$$

$$= i v_\perp J_1(-\sin\theta \hat{\mathbf{e}}_1 + \cos\theta \hat{\mathbf{e}}_2)$$

$$+ O(J_{\pm 1 \pm mL}), \quad m = 1,2,3,\ldots . \tag{38}$$

$$\nabla^2 \mathbf{A} - \frac{1}{v_A^2} \frac{\partial^2 \mathbf{A}_\perp}{\partial t^2} = -\frac{4\pi}{c} \mathbf{J}_{gc}, \tag{15}$$

$$\frac{d\nabla_\perp^2 \phi}{dt} + \frac{v_A^2}{c} (\mathbf{b}\cdot\nabla) \nabla^2 A_\parallel = 0. \tag{18}$$

$$\frac{1}{c} \frac{\partial A_\parallel}{\partial t} + \mathbf{b}\cdot\nabla\phi = 0. \tag{19}$$

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A generalized weight-based particle-in-cell simulation scheme

W.W. Lee *, T.G. Jenkins [1], S. Ethier

Princeton Plasma Physics Laboratory, Princeton University, Princeton, NJ 08543, United States

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ABSTRACT

A generalized weight-based particle simulation scheme suitable for simulating magnetized plasmas, where the zeroth-order inhomogeneity is important, is presented. The scheme is an extension of the perturbative simulation schemes developed earlier for particle-in-cell (PIC) simulations. The new scheme is designed to simulate both the perturbed distribution (δf) and the full distribution (full-F) within the same code. The development is based on the concept of multiscale expansion, which separates the scale lengths of the background inhomogeneity from those associated with the perturbed distributions. The potential advantage for such an arrangement is to minimize the particle noise by using δf in the linear stage of the simulation, while retaining the flexibility of a full-F capability in the fully nonlinear stage of the development when signals associated with plasma turbulence are at a much higher level than those from the intrinsic particle noise.

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Finite-β Simulation of Microinstabilities (DRAFT)

Edward A. Startsev and W. W. Lee

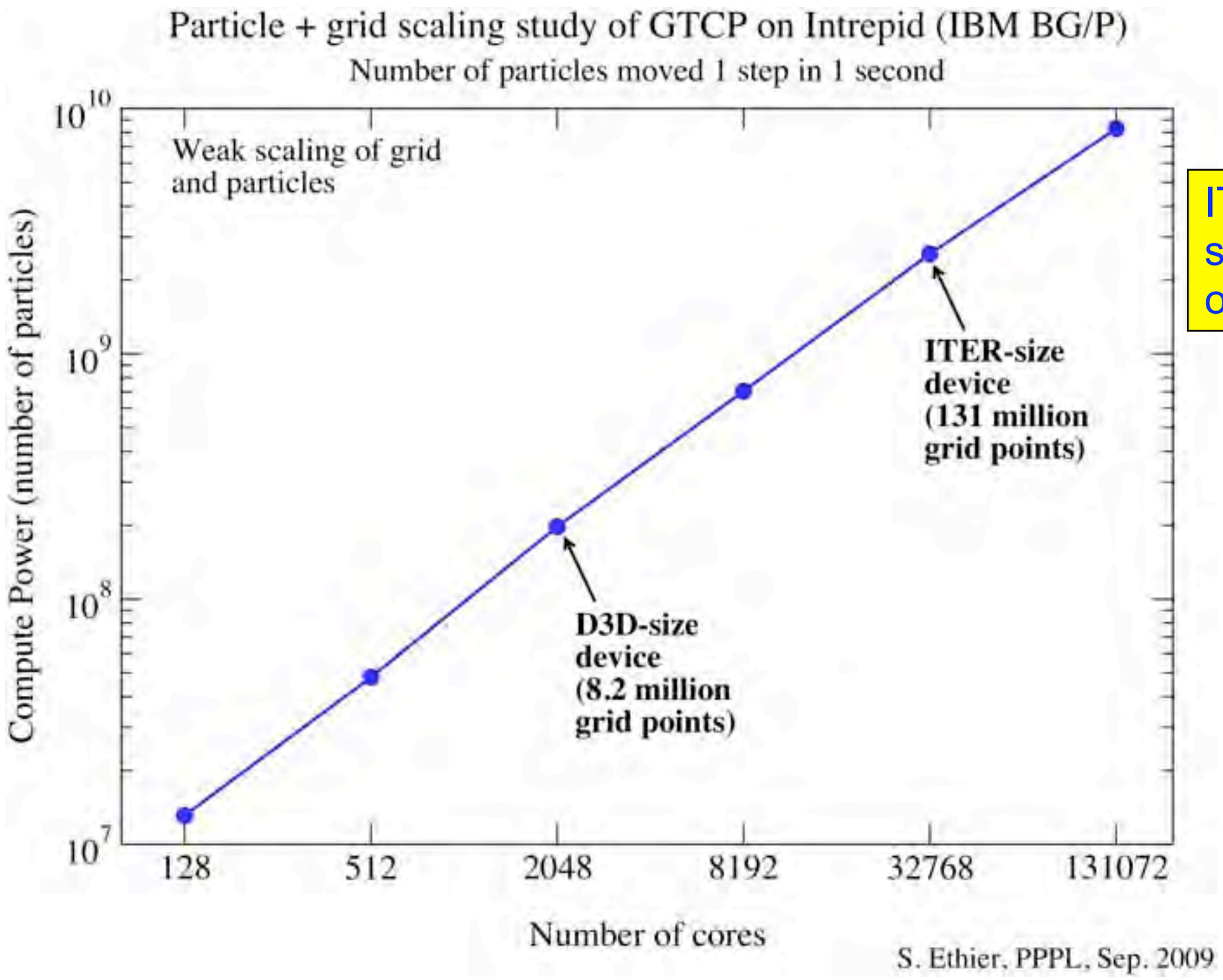
Princeton Plasma Physics Laboratory, Princeton University, Princeton, NJ 08543

(Dated: August 12, 2011)

Abstract

A new split-weight perturbative particle simulation scheme for finite-β plasmas in the presence of background inhomogeneities is presented. The scheme is an improvement over the original split-weight scheme [W. W. Lee *et. al*, Phys. Plasmas **8**, 4435 (2001)], which splits the perturbed particle response into adiabatic and non-adaibatic parts. In the new scheme, by further separating out the non-adaibatic response of the particles associated with the quasi-static bending of the magnetic field lines in the presence of background inhomogeneities of the plasma, we are able to demonstrate the finite-β stabilization of drift waves and ion temperature gradient modes using a simple gyrokinetic particle code based on realistic fusion plasma parameters. However, for $\beta m_i/m_e \gg 1$, it becomes necessary to use the electron skin depth as the grid size of the simulation to achieve accuracy in solving the resulting singularly perturbed equations. The proposed scheme is most suitable for studying shear-Alfvén physics in general geometry using straight field line coordinates for microturbulence and magnetic reconnection problems.

Scaling of 2D Domain Decomposition on Blue Gene/P System



ITER-size simulation with only 512MB/core!

Princeton Plasma Physics Laboratory

Co-Design Center for Exascale Simulations of Fusion Plasmas

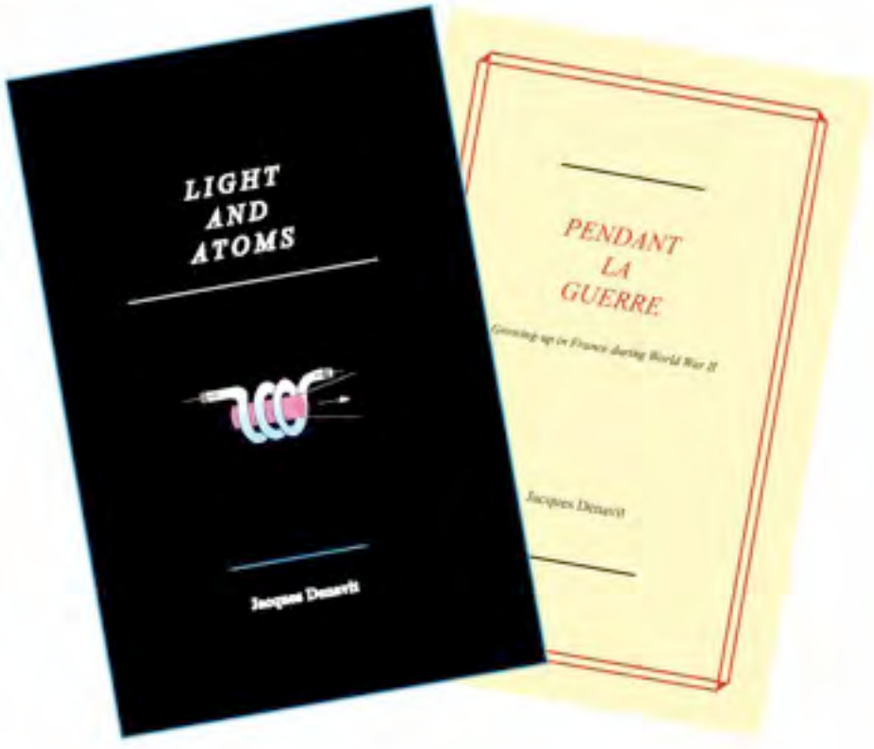
W. W. Lee (PI), PPPL, Princeton, NJ 08543 (609-243-2647; wwlee@pppl.gov)
Z. Lin (UC Irvine); G. Chiu (IBM); B. F. Smith (ANL); E. Strohmaier (LBNL); C. Kamath (LLNL);
D. Spong (ORNL); R. Lucas (USC); K. L. Ma (UC Davis); V. Decyk (UCLA); A. Sanderson (U. Utah)

The capability of simulating realistic fusion plasmas is an important step in the design and operation of experimental thermonuclear devices, such as ITER, presently under construction in France, with the United States as a major partner. For the success of ITER tokamak experiments, it is critical that we understand, predict, and control turbulent transport in fusion plasmas, so that the high-pressure plasmas remain confined and are self-sustained by the heating from the fusion reaction.

An important tool for understanding plasma turbulence in magnetic fusion research is the gyrokinetic Particle-In-Cell (PIC) simulation, which started in the 1980s at PPPL. PIC codes solve the equations of motion for the particles, and the associated field equations, in three-dimensions (instead of the usual phase space in 5~6 dimensions). As these equations are essentially linear, they are amenable to the complicated three-dimensional toroidal geometry of the tokamak, and the resulting physics is nearly two-dimensional. This allows us to naturally express the parallelism and locality in the problem, making these codes adaptable to modern-day, multi-core, massively parallel computers. When combined with the advances in gyrokinetic theory and algorithms, and the computational power of the proposed exascale systems, PIC codes make it a real possibility that we can realistically simulate an ITER burning plasma in one code with comprehensive physics using a single computing platform.

Unfortunately, this proposal was rejected by
US DoE ASCR in July 2011

Acknowledgement



- I'm thankful that I was introduced to this field by my advisor, Prof. Jacques Denavit, who has just finished a book -- Light and Atom by DeltaT Books (2011)

- I'm grateful that so many pioneers in our field laying the ground work for us, especially, Prof. Dawson.

- I've benefitted greatly by working with many of my colleagues at PPPL, past and present, and not only those that I mentioned during the talk.

- I would like to thank PPPL for creating the environment that is conducive to innovative work