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Collisional effects on electron beam transport in dense plasma

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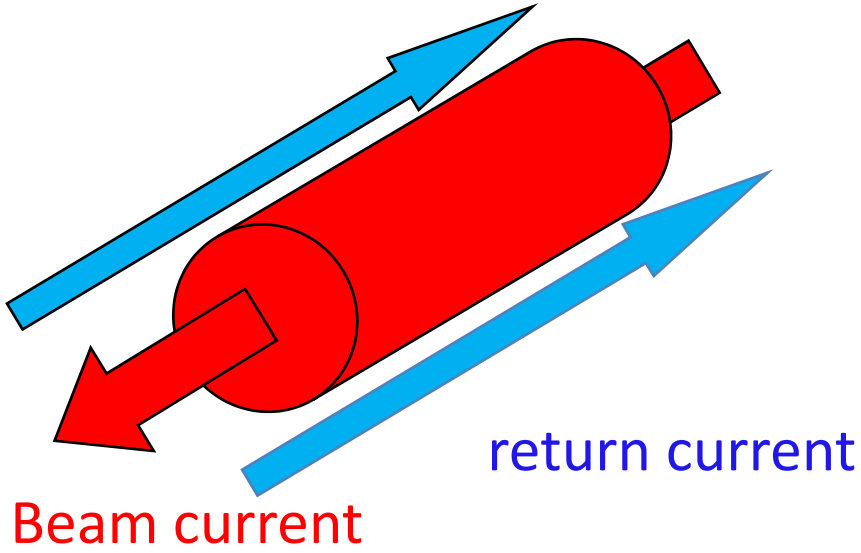
Outline

- Motivation of the study
- Dispersion relation of beam-plasma instabilities in full k-space
- Collisional effects
- Effects of background plasma temperatures on filamentation and TS instabilities
- Summary

Beam-plasma interaction

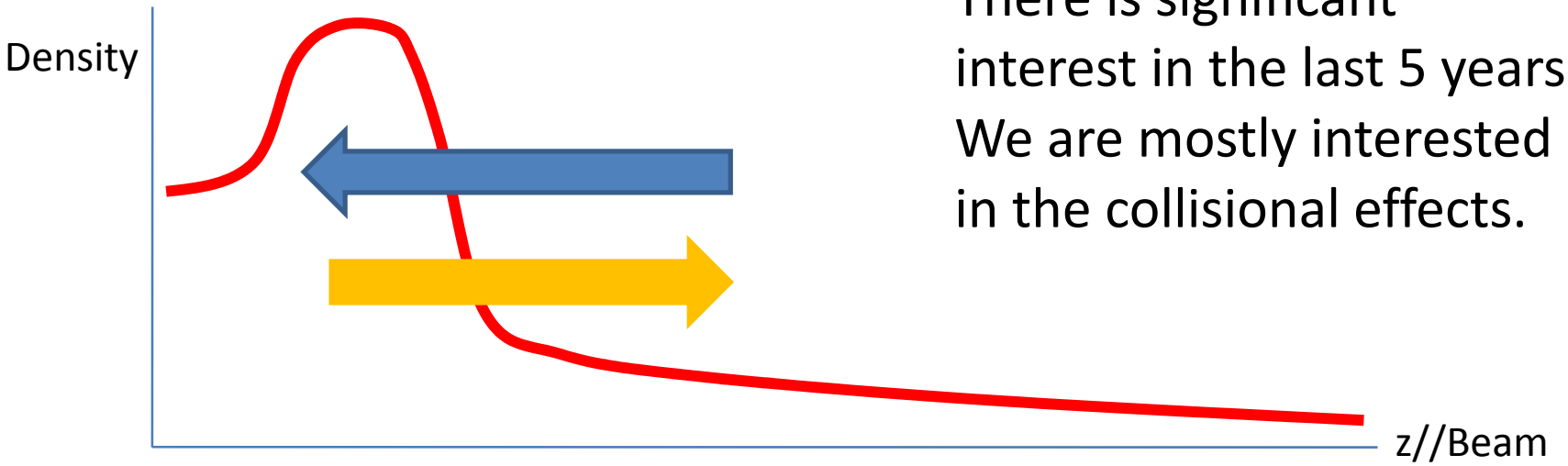
Beam-plasma interaction is an fundamental problem, which is relevant with fast ignition of fusion targets, some astrophysical problems, such the origin of cosmic rays, magnetic fields etc.

A typical system: when an electron beam transports in plasma, a return current is induced: A. R. Bell et al. PPCF 39, 653 (1997) :

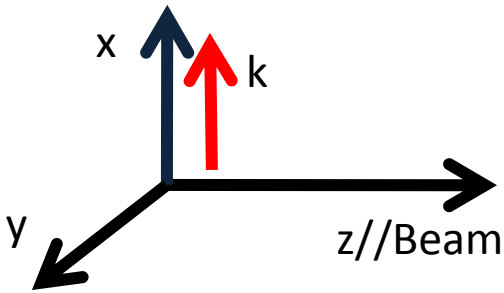


Because this system has high free energy – the system is far from thermal equilibrium , it is unstable.

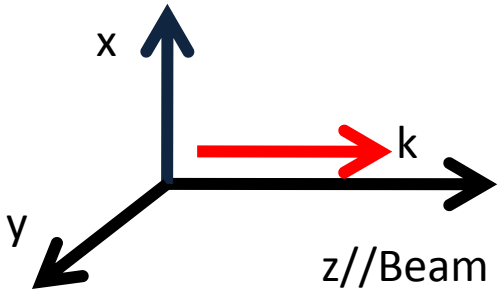
Microscope instabilities in beam transport



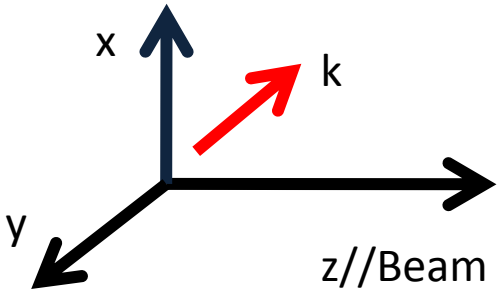
There is significant interest in the last 5 years. We are mostly interested in the collisional effects.



Filamentation Instab.



Two-stream Instab.



Hybrid Instab.

Brief History

- 1925, Langmuir found the presence of plasma oscillations in a beam-plasma system [1]；
- 1948, Pierce proposed the presence of unstable plasma oscillations in this kind of system [2]；
- 1949, Bohm et al. considered the two-stream instability [3]；
- 1959, Weibel found the electromagnetic instability in anisotropic thermal plasma [4];
- 1959, Fried found the beam filamentation instability [5]；
- 1960, Bludman et al. found the oblique mode instability in fluid model [6]；
- 2008, Bret et al. formulated the kinetic theory in full k-space, but limited to some special cases and without collisional effects [7]； Kong and Ren formulated the theory with the waterbag model [8].

[1] L. Langmuir, Phys. Rev. 26, 585 (1925)
[2] J. R. Pierce, J. Appl. Phys. 19, 231 (1948)
[3] D. Bohm and E. P. Gross, Phys. Rev. 75, 1851 (1949)
[4] E. S. Weibel, Physical Review Letters 2, 83 (1959)
[5] B. D. Fried. Phys, Fluids 2, 337 (1959)
[6] S. A. Bludman et al, Phys. Fluids 3, 747 (1960)
[7] A. Bret et al, Phys. Rev. Lett. 100, 205008 (2008)
[8] X. Kong et al., Phys. Plasmas 16，032107 (2008)

Our model

- Current neutralization (A. R. Bell et al. PPCF 39, 653 (1997)):

$$n_b v_{db} + n_p v_{dp} = 0$$

Three time scales:

$$\tau_e = \frac{1}{4\pi\sigma},$$

Relaxation time

$$\tau_m = \frac{4\pi\sigma r_b^2}{c^2},$$

Magnetic diffussion

$$\tau_p$$

Electron beam duration

Our model

- Current neutralization (A. R. Bell et al. PPCF 39, 653 (1997)):

$$n_b v_{db} + n_p v_{dp} = 0$$

In fast ignition: $\tau_e << \tau_p << \tau_m$

→ Current neutralization approximation is applicable and macro-instabilities can be ignored.

- Collisional model: Krook operator:

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}_\alpha \frac{\partial f_\alpha}{\partial \vec{r}} + q_\alpha \left(\vec{E} + \frac{\vec{v}_\alpha}{c} \times \vec{B} \right) \frac{\partial f_\alpha}{\partial \vec{p}_\alpha} = -\nu_\alpha \left(f_\alpha - f_{\alpha 0} \right)$$

Dispersion relation in full k-space (without collisions)

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}_\alpha \frac{\partial f_\alpha}{\partial \vec{r}} + q_\alpha \left(\vec{E} + \frac{\vec{v}_\alpha}{c} \times \vec{B} \right) \frac{\partial f_\alpha}{\partial \vec{p}_\alpha} = 0$$

$$\nabla \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \sum_\alpha \vec{j}_\alpha,$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t},$$

$$\vec{j}_\alpha = q_\alpha n_\alpha \vec{v}_\alpha$$

Dispersion relation in full k-space

$$\det\left|\frac{\omega^2}{c^2}\varepsilon_{ij}+k_ik_j-k^2\delta_{ij}\right|=0,$$

where:

$$\varepsilon_{ij}=\delta_{ij}+\sum_\alpha\frac{\omega_{p\alpha}^2}{n_\alpha\omega^2}\int d^3\vec{p}_\alpha\frac{\left[m_\alpha\gamma_\alpha^*\omega-\vec{k}\cdot\vec{p}_\alpha\right]\delta_{lj}+k_lp_{\alpha j}}{\gamma_\alpha^*\left[m_\alpha\gamma_\alpha^*\omega-\vec{k}\cdot\vec{p}_\alpha\right]}p_{\alpha i}\frac{\partial f_\alpha\left(\vec{p}_\alpha\right)}{\partial p_{\alpha l}},$$

Oblique mode:

$$\left(\omega^2\varepsilon_{zz}-k_x^2c^2\right)\left(\omega^2\varepsilon_{xx}-k_z^2c^2\right)=\left(\omega^2\varepsilon_{xz}-k_xk_zc^2\right)\left(\omega^2\varepsilon_{zx}-k_xk_zc^2\right)$$

Filamentation mode: $\left(\omega^2\varepsilon_{zz}-k_x^2c^2\right)\varepsilon_{xx}=\omega^2\varepsilon_{xz}\varepsilon_{zx}$

Two-stream mode: $\varepsilon_{zz}=0$

Simplification to calculate the diaelectric tensor

To solve the dispersion relation, one needs to solve the plasma dispersion

$$Z(\xi) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{e^{-t^2}}{t-\xi} dt$$

For the relativistic Maxwell-Jüttner distribution，

$$f_\alpha^0(\mathbf{p}) = \frac{\mu_\alpha}{4\pi\gamma_\alpha^2 K_2(\mu_\alpha/\gamma_\alpha)} \exp[-\mu_\alpha(\gamma(\mathbf{p}) - \beta_\alpha p_y)],$$

$$K_2(\xi) = \int_0^\infty \exp(-\mu_\alpha \cosh\xi) \sinh^2\xi \cos h\xi d\xi,$$

The longitudinal and transverse momenta are coupled through the Lorentz factor, making the dispersion function even hard to solve.

Expanding the distribution function in terms of drifting momentum, one obtains the drift Maxwellian distribution

$$f_{0b}(\vec{P}_b) = \frac{n_{0b}}{2\pi m\gamma_b^2 (T_{tb}T_{lb})^{1/2}} e^{-P_x^2/2m\gamma_b T_{tb}} e^{-(P_z-P_{db})^2/2m\gamma_b^3 T_{lb}},$$

$$f_{0b} = \frac{n_b}{2\pi m \gamma_b^2 \left(T_{tb} T_{lb}\right)^{1/2}} \exp\left[-\frac{p_x^2}{2m\gamma_b T_{tb}} - \frac{(p_z - p_{db})^2}{2m\gamma_b^3 T_{lb}}\right]$$

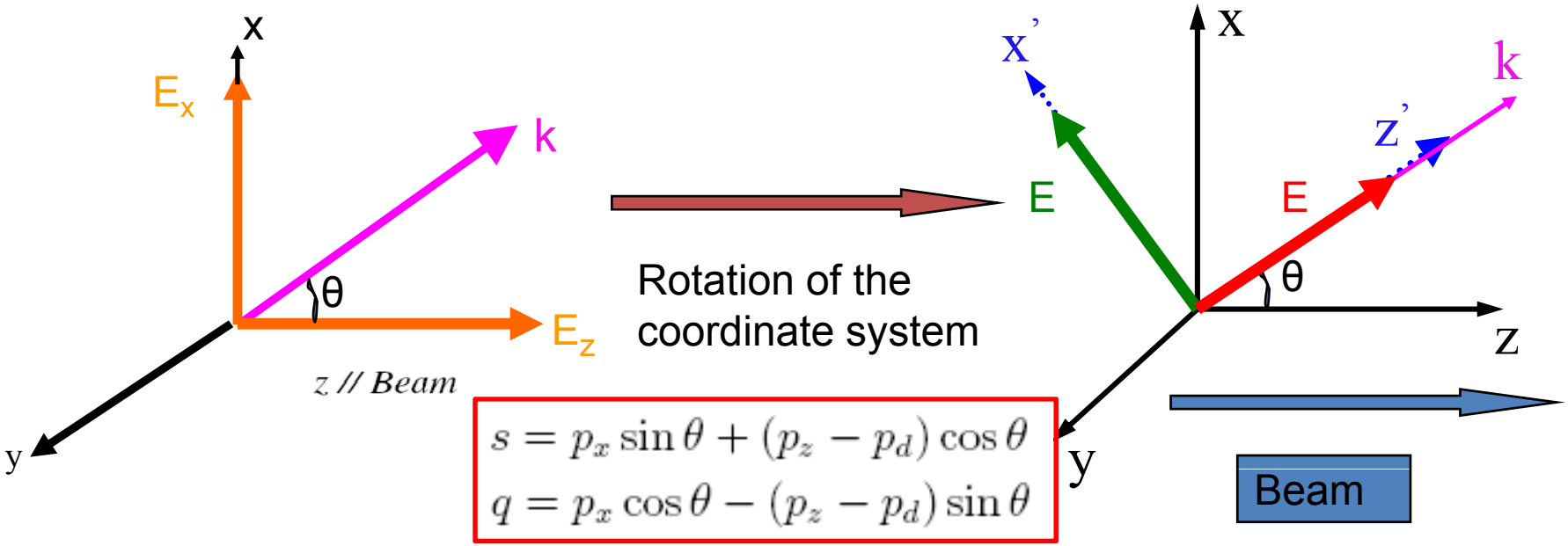
Momentum spread and temperatures are not synchronously isotropic. For example, $T_{tb} = T_{lb} = T_b$

$$\begin{aligned} p_{lb} &= \frac{1}{n_b} \int_{-\infty}^{\infty} dp_x \int_0^{\infty} (p_z - p_{db}) f_{0b} dp_z \\ &= \sqrt{\gamma_b^3 m T_b}, \\ p_{tb} &= \sqrt{\gamma_b m T_b}. \end{aligned}$$

Mathematical transform—rotating coordinate system

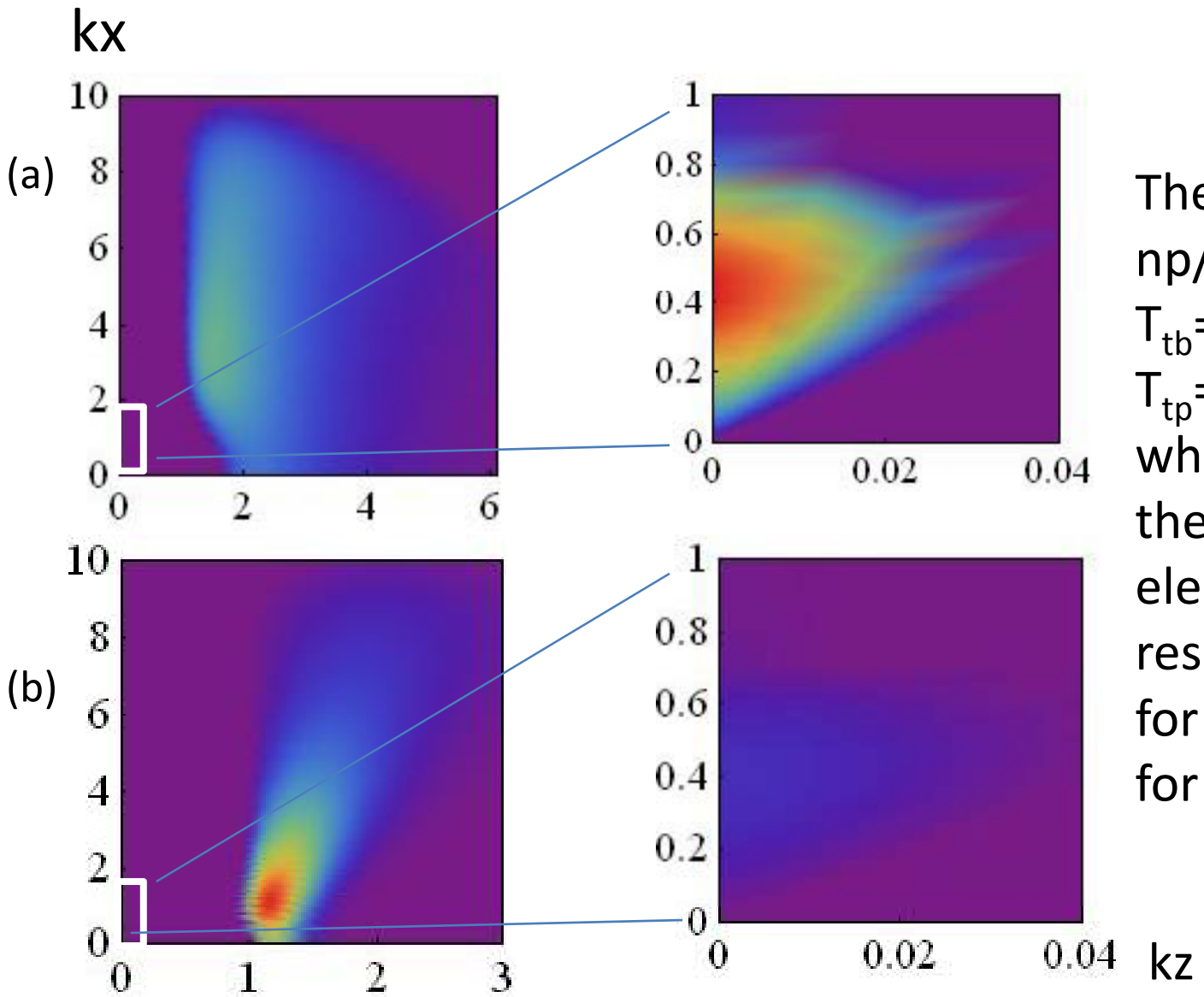
With the drift Maxwellian distribution, one can solve the dispersion relation for filamentation instability and two-stream instability. But for oblique instability, it is not enough--- the transverse and longitudinal momenta are coupled each other again.

Finally to solve the dispersion relation, the plasma dispersion function is approximated by polynomial Pade approximation.



$$s = p_x \sin\theta + (p_z - p_d)\cos\theta$$
$$q = p_x \cos\theta - (p_z - p_d)\sin\theta$$

Example results



The parameters are np/nb=30, γ_b=4.9, T_{tb}=30 keV, and T_{tp}=T_{lp}=250 eV, where nb and np are the beam and plasma electron density, respectively. T_{lb}=T_{tb} for (a) andT_{lb}=$\gamma_b^{-2}T_{tb}$ for (b).

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History about the collisional effects

◆ Three papers：fluid model for the background plasma, and contradict conclusions between them.

(i). K. Molvig, PRL 35,1504 (1975)

(ii). Anupam et al, PRL 101, 255001 (2008)

} enhanced

(iii). M. Honda, PRE 69, 016401 (2005): suppressed

◆ In 1960, Bludman et al. considered collisional effect based upon fluid model, ignore the electromagnetic component of the oblique mode.[1]

[1] S. A. Bludman et al, Phys. Fluids 3, 747 (1960)

◆Hybird simulation with Ohm's law for the background plasma response, excludes the oblique mode completely.[2,3]

[2] J. J. Honrubia, and J. Meyer-ter-Vehn, Nucl. Fusion 46, L25 (2006)
[3] M. Storm et al, Phys. Rev. Lett. 102, 235004 (2009)

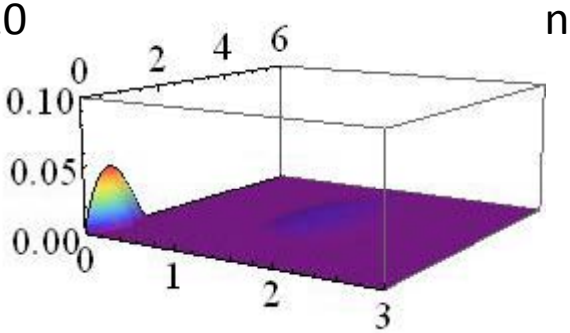
Similar dispersion relation

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}_\alpha \frac{\partial f_\alpha}{\partial \vec{r}} + q_\alpha \left(\vec{E} + \frac{\vec{v}_\alpha}{c} \times \vec{B} \right) \frac{\partial f_\alpha}{\partial \vec{p}_\alpha} = -\nu_\alpha \left(f_\alpha - f_{\alpha 0} \right)$$

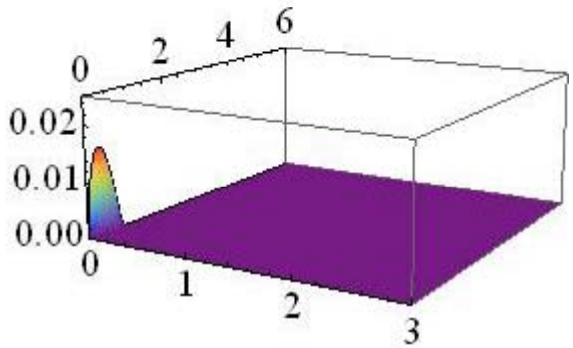
$$\det \left| \frac{\omega^2}{c^2} \varepsilon_{ij} + k_i k_j - k^2 \delta_{ij} \right| = 0,$$

$$\varepsilon_{ij} = \delta_{ij} + \sum_\alpha \frac{\omega_{p\alpha}^2}{n_\alpha \omega^2} \int d^3 \vec{p}_\alpha \frac{\left[m_\alpha \gamma_\alpha^* \omega - \vec{k} \cdot \vec{p}_\alpha \right] \delta_{lj} + k_l p_{\alpha j}}{\gamma_\alpha^* \left[m_\alpha \gamma_\alpha^* \left(\omega + i\nu_\alpha \right) - \vec{k} \cdot \vec{p}_\alpha \right]} p_{\alpha i} \frac{\partial f_\alpha \left(\vec{p}_\alpha \right)}{\partial p_{\alpha l}},$$

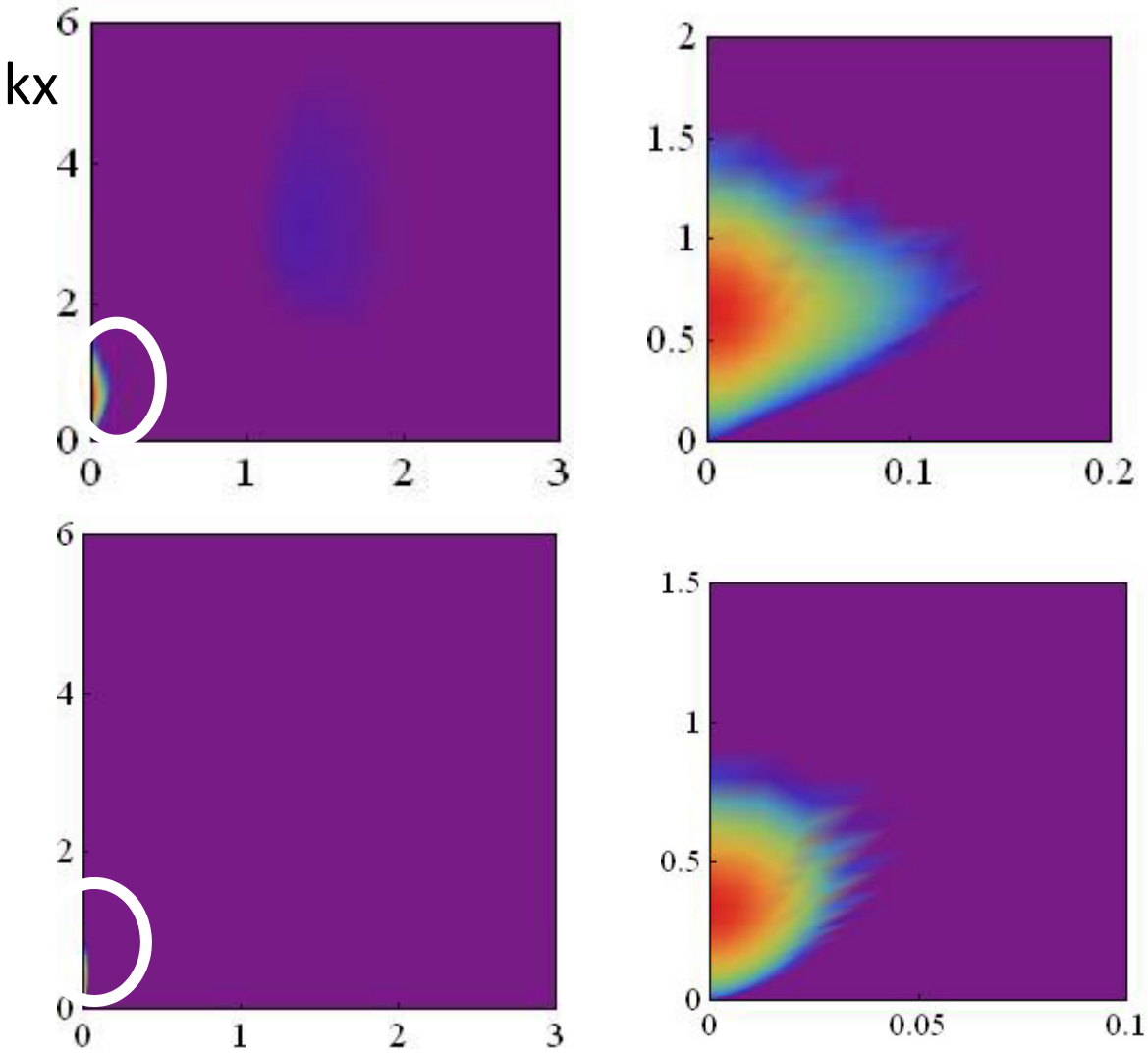
np/nb=10



np/nb=30



Collisions reduce oblique mode, enhance the filamentation instability

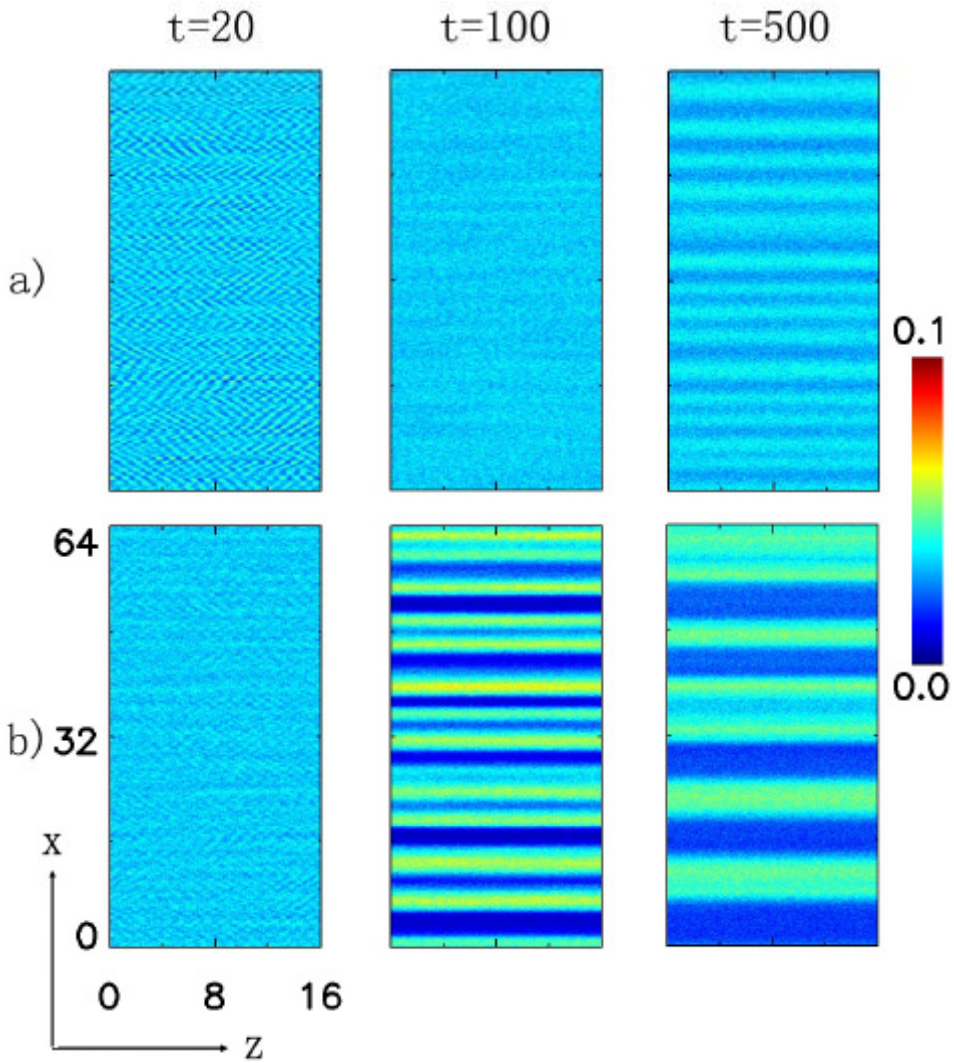


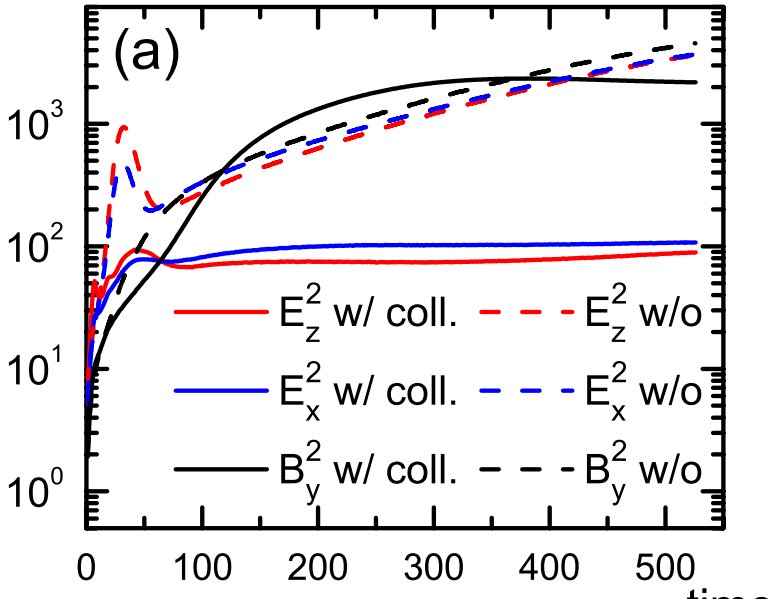
$$\gamma_b = 4.9$$

$$T_{lb} = T_{tb} = 30 \text{ keV}$$

$$T_{tp} = T_{lp} = 250 \text{ eV}$$

Comparison of PIC simulation





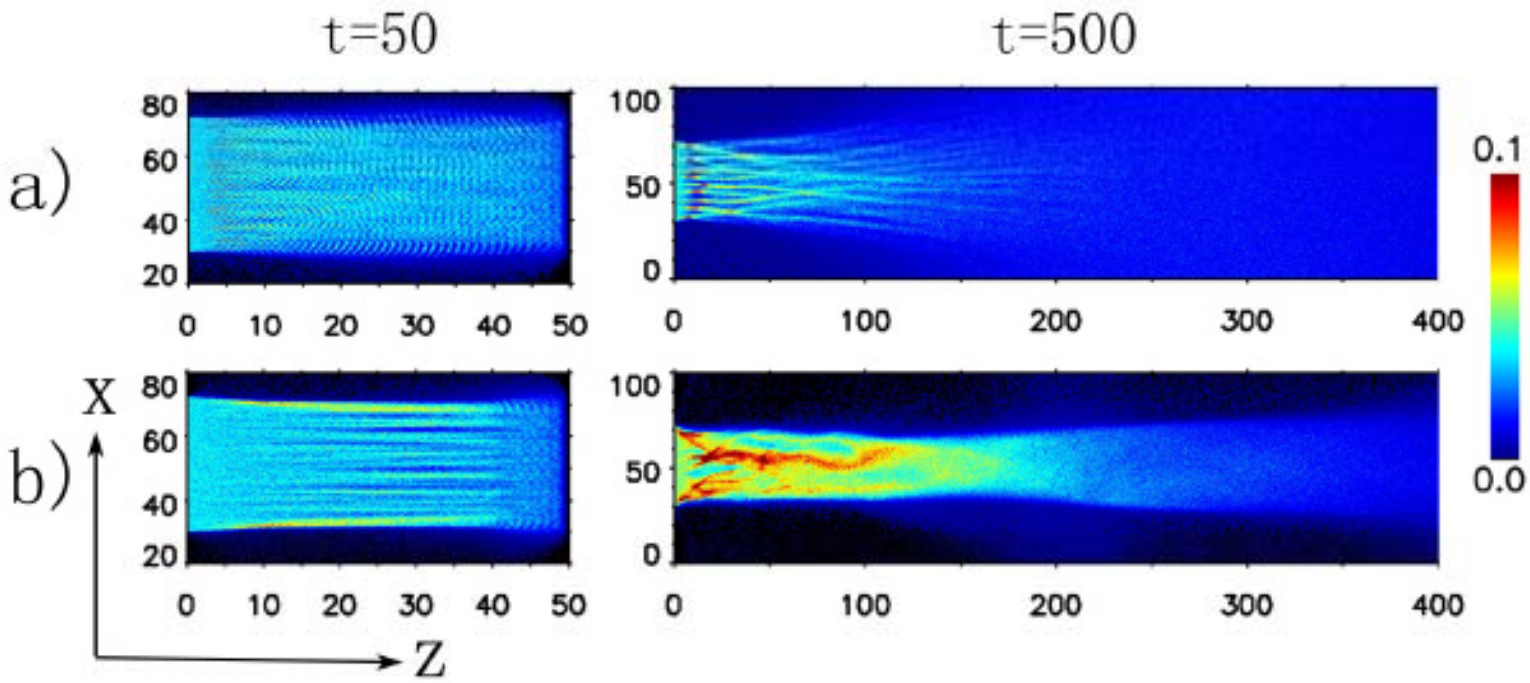
For solid density plasma, there is no net increase of the magnetic fields at the saturation level.

Periodic boundaries

B. Hao et al., submitted (2011)

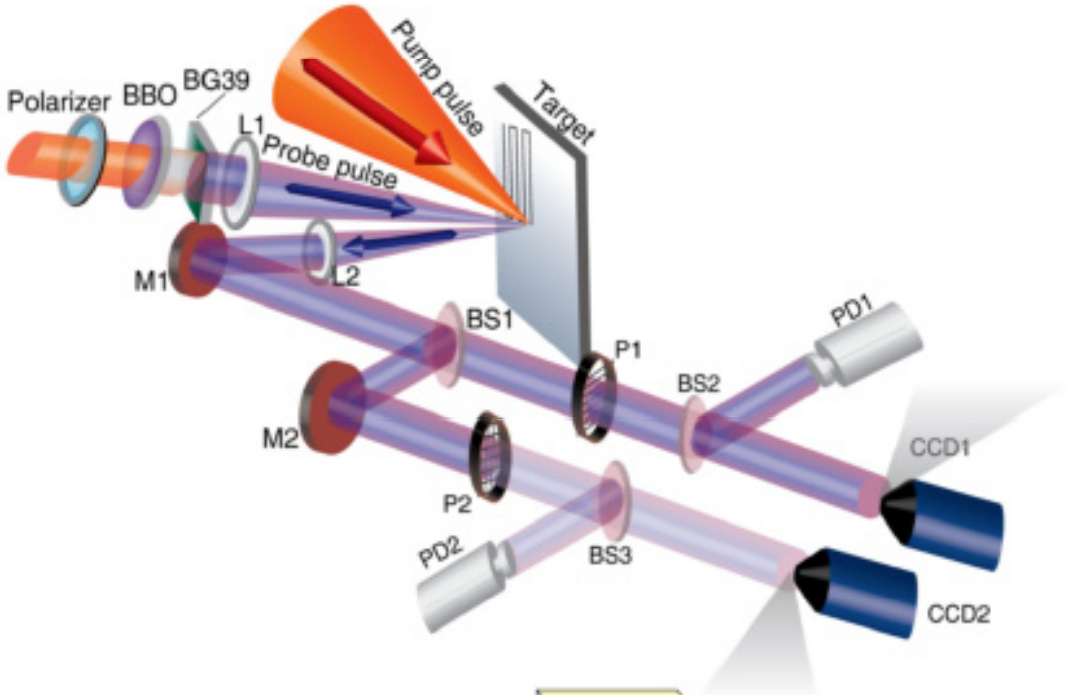
Comparison of PIC simulation

Open boundary in the z-direction

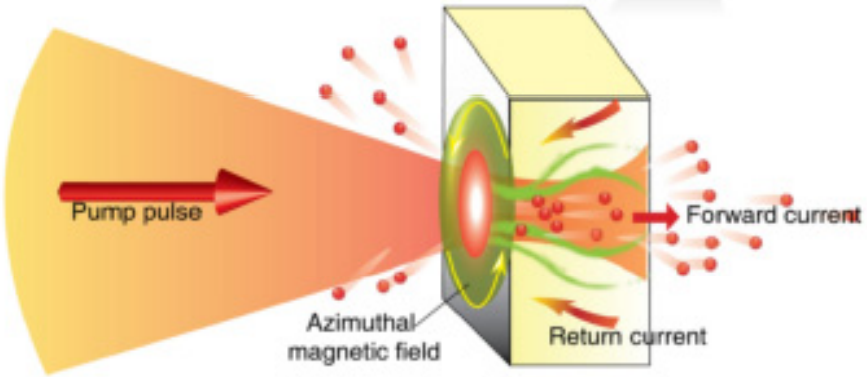


Because of the collisional effect, the electrostatic mode is suppressed, the electromagnetic mode is enhanced, the resulting quasi-static magnetic fields help to guide electron beams, which is beneficial for the fast ignition scheme.

Measurement of quasi-static magnetic fields produced in laser solid interaction

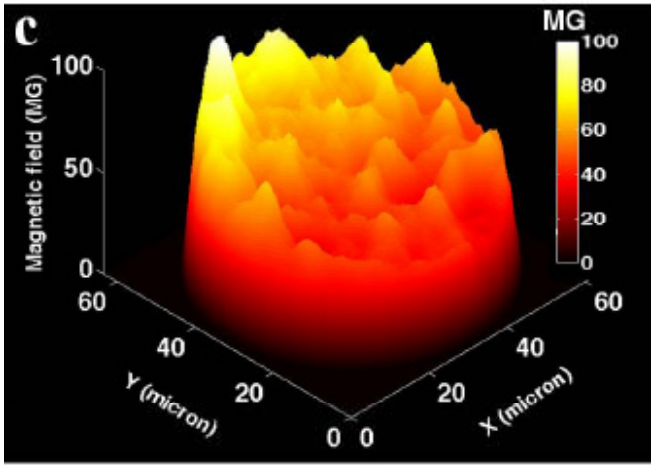


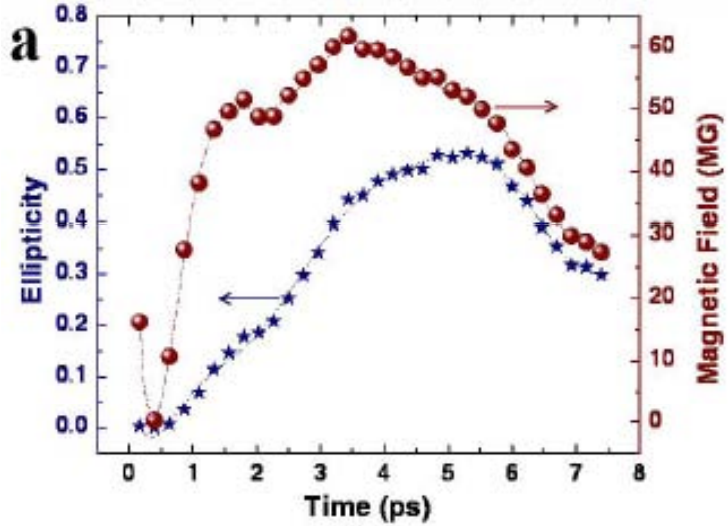
The probe laser pulse is at $2w_0$.

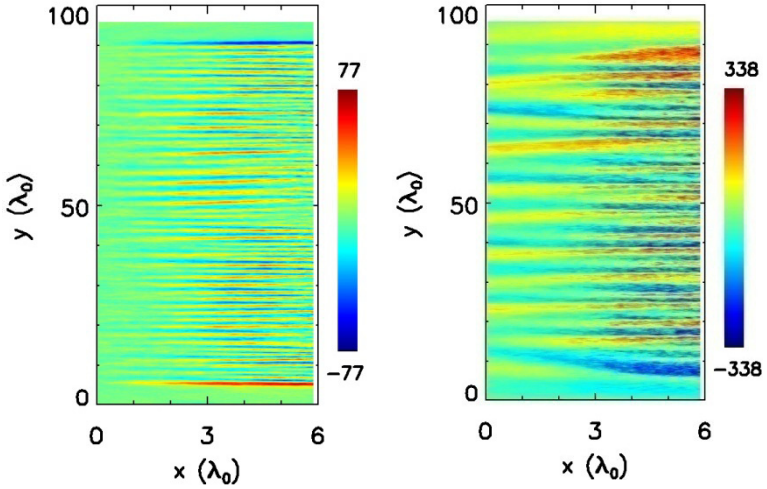


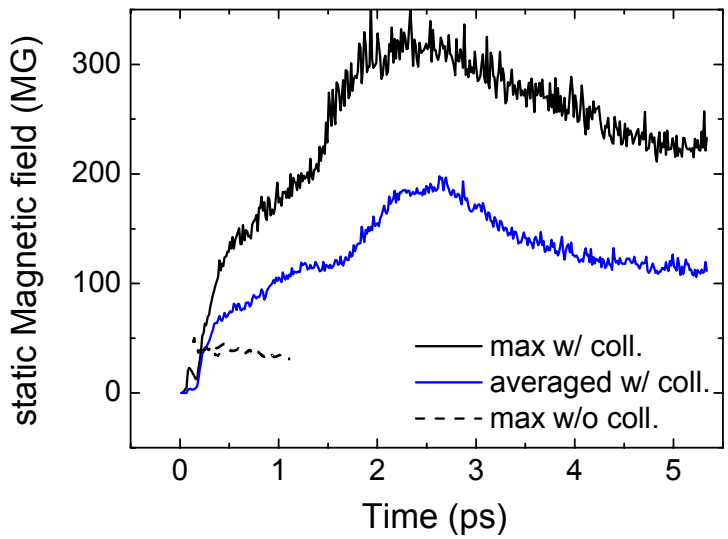
R. Kumar (TIFR, India)

Simulation also shows enhanced B field generation due to the collisional effect









R. Kumar (TIFR, India), W.J. Ding et al. (IoP-CAS)

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Relevant work

- **Earlier studies mainly consider the beam temperature effect, the effect of background plasma is largely ignored. The beam temperature usually tends to suppress the filamentation instability (space charged effects are also ignored);**

- **Earlier kinetic studies use waterbag model of the distribution function, which allows easy treatment of the plasma dispersion function. But in this case, filamentation instability can be suppressed completely[1,2];**

- **So far the backgroud plasma temperature has not yet received sufficient attention. In Fast ignition, the background plasma may be heated to keV level after pre-compression.**

[1] L. O. Silva et al, Phys. Plasmas 9, 2458 (2002)
[2] A. Karmakar et al, Phys. Rev. Lett. 101, 255001 (2008)

Dispersion relation for filamentation instability

$$\left(\omega^2\varepsilon_{zz} - k_x^2c^2\right)\varepsilon_{xx} = \omega^2\varepsilon_{xz}\varepsilon_{zx}$$

Therefore , generally the filamentation instability is coupled with the electrostic osillation through ε_{xz}

$$E_x = -\varepsilon_{xz}/\varepsilon_{xx}E_z$$

When ignoring the transverse electric field Ex=0, ε_{xz}=0

$$\omega^2\varepsilon_{zz} - k_x^2c^2 = 0$$

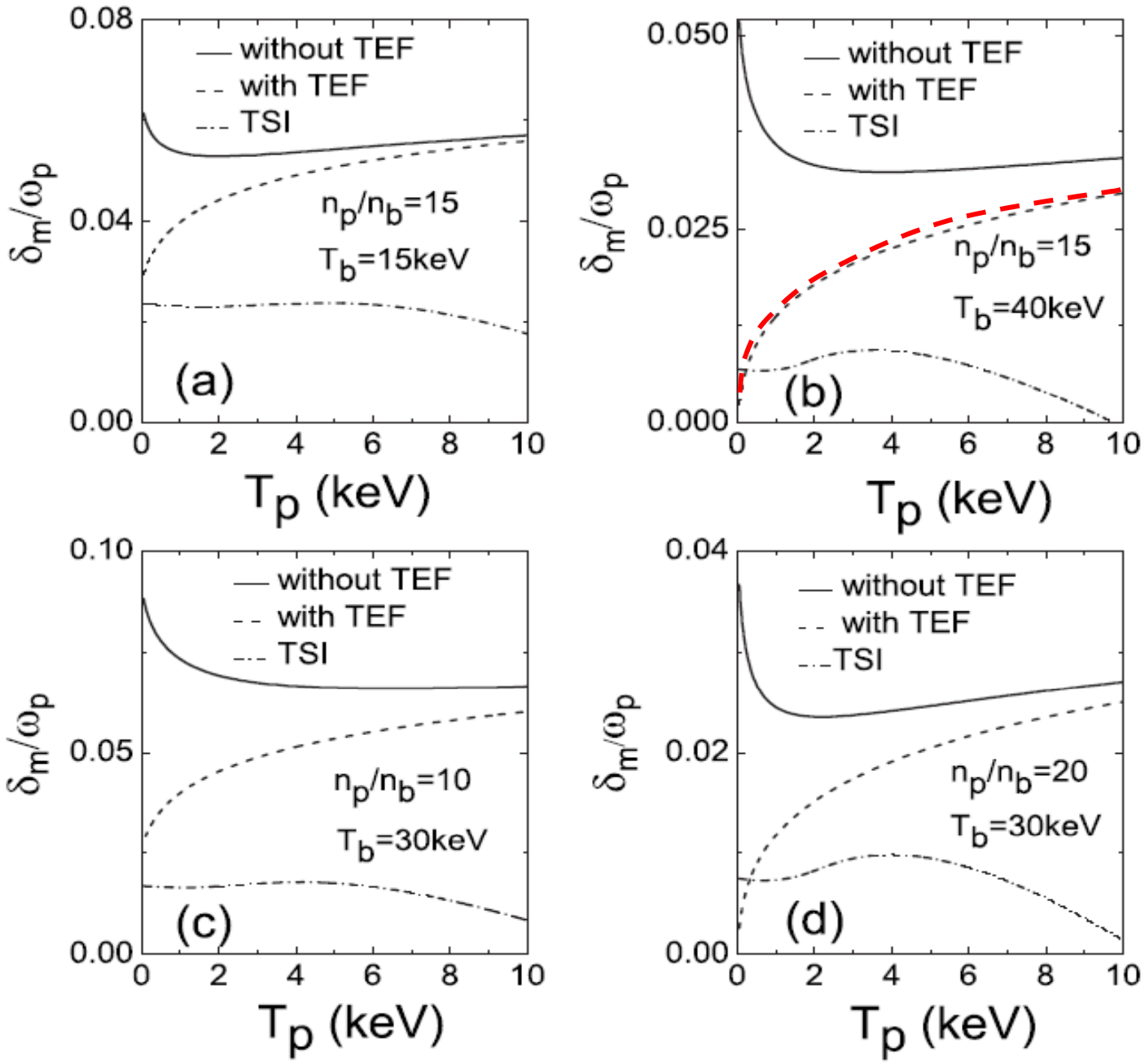
Effect of background plasma temperature on FI and TS instabilities

- In contrast with the beam temperature effect, the filamentation instability increases with the background temperature.

$$\delta \sim \sqrt{\frac{2}{\pi}} k \frac{\omega_b^2 v_{db}^2 m/T_b + \omega_b^2 \left(\gamma_b - 1/\gamma_b^3\right) - k^2 c^2}{\omega_p^2 \left(v_{dp}^2 + T_p/m\right)} \left(\frac{T_p}{m}\right)^{3/2}$$

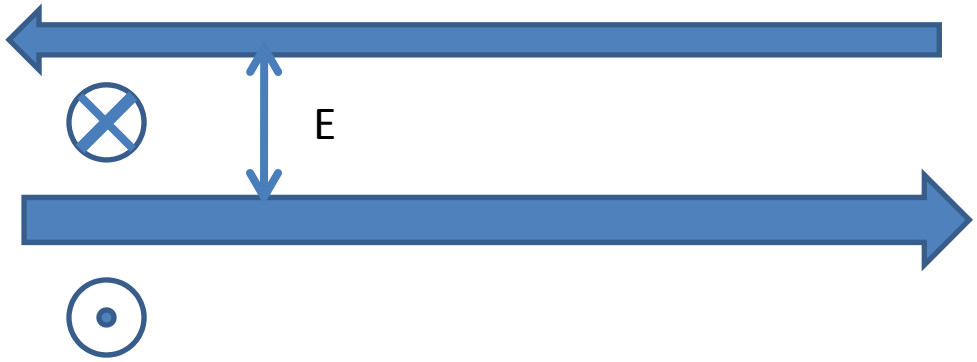
$$\delta_m \sim \frac{2\sqrt{6}}{9\sqrt{\pi}} \frac{\left[\omega_b^2 v_{db}^2 m/T_b + \omega_b^2 \left(\gamma_b - 1/\gamma_b^3\right)\right]^{3/2}}{\omega_p^2 \left(v_{dp}^2 + T_p/m\right) c} \left(\frac{T_p}{m}\right)^{3/2}$$

B. Hao et al., PRE (2009)



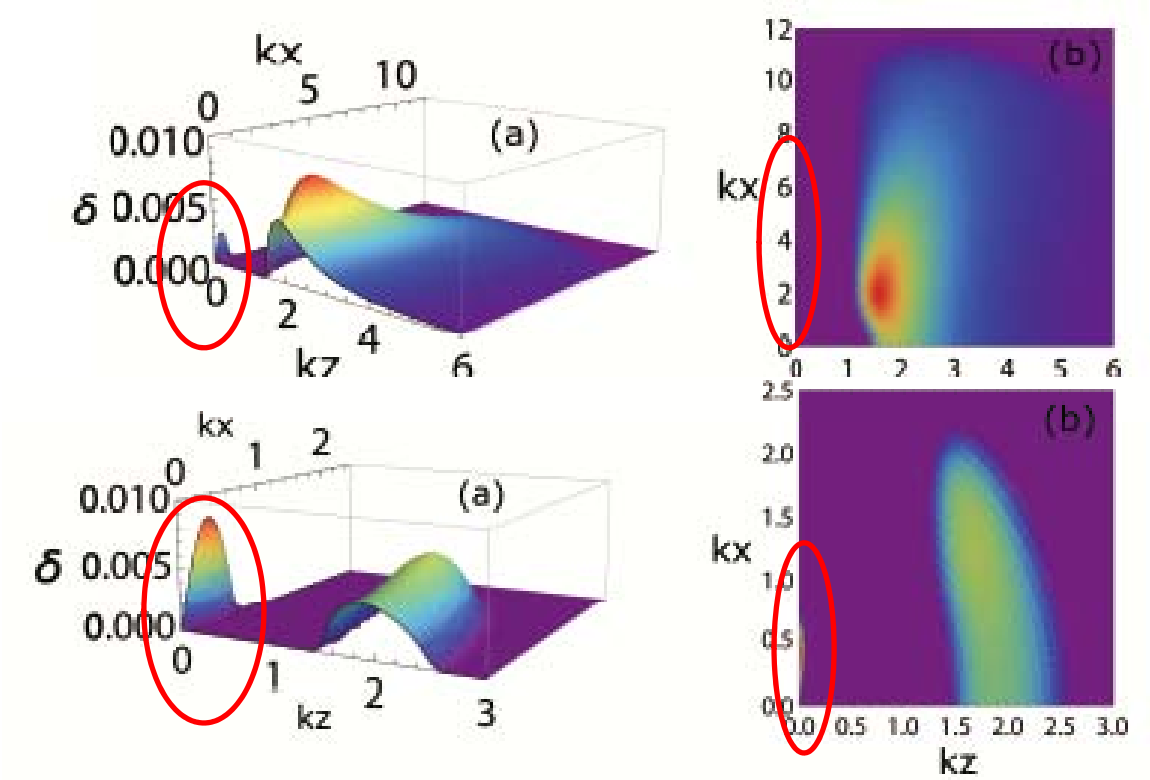
B. Hao et al., PRE (2009)

The forward beam current and the returning plasma current will pinch at first, resulting in space charge separation . The electrostatic fields tend to balance the magnetic pinch force and inhibit further filamentation. As the plasma temperature grows, the thermal suppression of the electrostatic fields becomes stronger, as a whole the CFI is enhanced as the plasma temperature grows.



Effect of background electron temperatures in full k-space

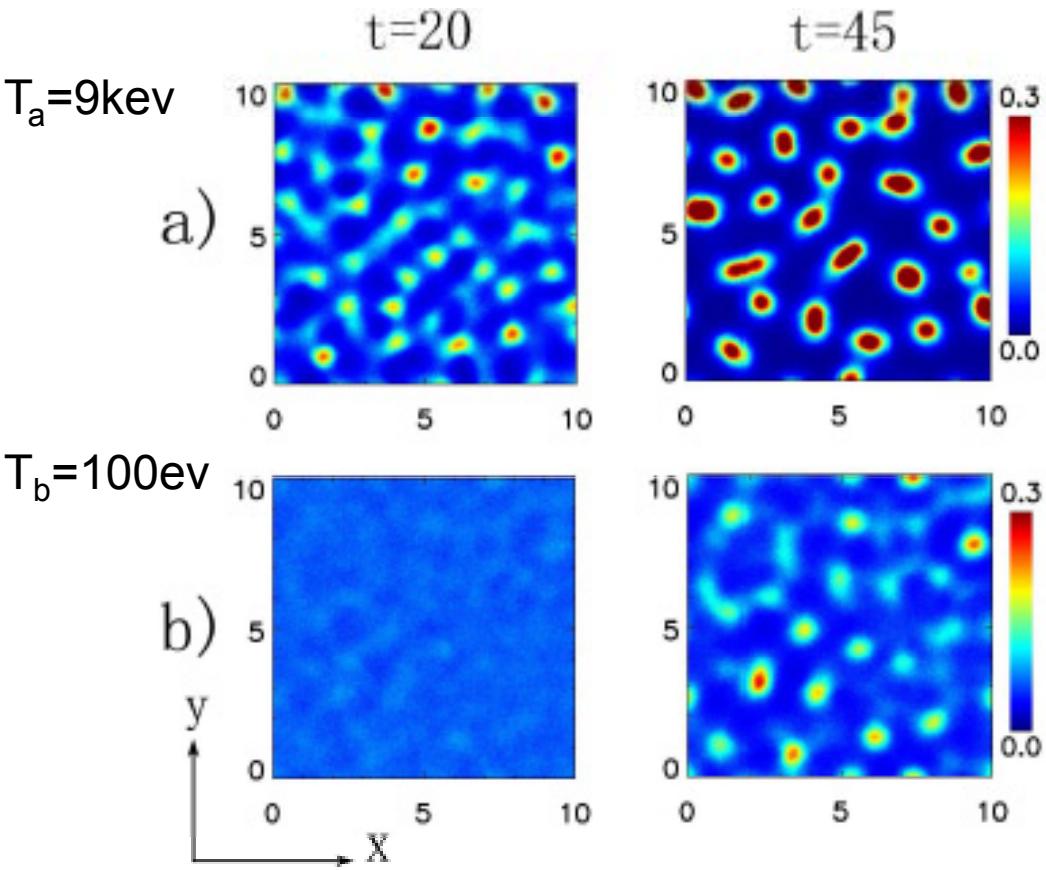
- The growth rate of the ES component decrease with Tp, and the that of the EM component increases with Tp.



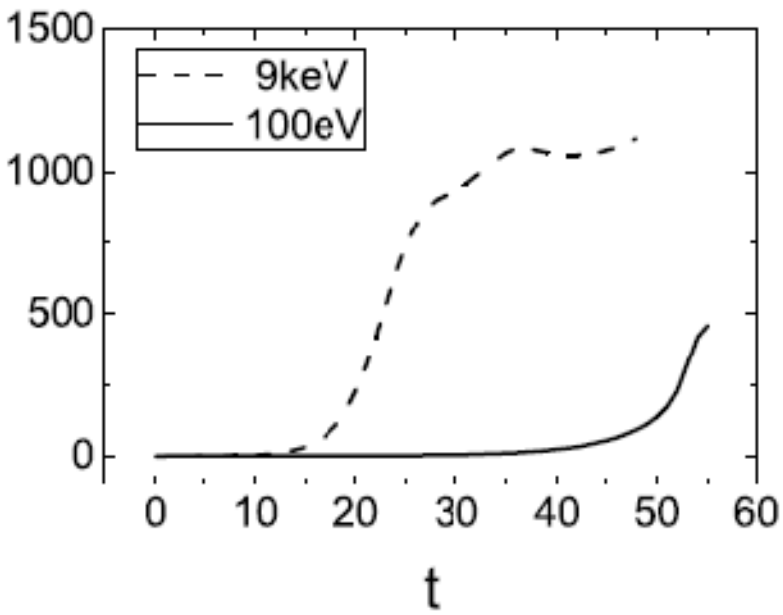
T_P=200eV

T_P=5 keV

PIC Simulation Test



2D PIC result



B. Hao et al., PRE (2009)

The electron density is higher at high temperature, and it may have important effect on the saturation level of the self-generated B fields and turbulance formation.

Summary

- Dispersion relation for kinetic beam-plasma instabilities in full k-space is solved by rotating the coordinate system, as well as by use of the Padé approximation of the plasma dispersion function. It can be used to solve the system instabilities for various beam-plasma parameters.

- Generally the collisional effect enhances the EM component and suppress the ES component, and the filamentation instability becomes the most unstable mode in the parameters studied.

- The growth rates of filamentation instability and the EM component of the oblique instability are found to increases anomalously with the background plasma temperature. This is caused by the suppression of the electrostatic field component at high background plasma temperatures.

Thank you for your attention!